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CONDITIONAL FRAGILITY FUNCTIONS AND THE UNCERTINTY IN VULNERABILITY ASSESSMENT OF BUILDING PORTFOLIOS

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Abstract

State-of-the-art methods for the assessment of building fragility consider the structural capacity and seismic demand variability in the estimation of the probability of exceeding different damage states. However, questions remain regarding the appropriate treatment of such sources of uncertainty from a statistical significance perspective. In this study, material, geometrical and mechanical properties of a number of building classes are simulated by means of a Monte Carlo sampling process in which the statistical distribution of the aforementioned parameters is taken into consideration. Record selection is performed in accordance with hazard-consistent distributions of a comprehensive set of intensity measures, and issues related with *sufficiency, efficiency, predictability* and *scaling robustness* are addressed. Based on the appraised minimum number of ground motion records required to achieve statistically meaningful estimates of response variability conditioned on different levels of seismic intensity, the concept of *conditional fragility functions* is presented. These functions translate the probability of exceeding a set of damage states as a function of a secondary *sufficient* intensity measure, when records are selected and scaled for a particular level of primary seismic intensity parameter. It is demonstrated that this process allows a hazard-consistent and statistically meaningful representation of uncertainty and correlation in the estimation of intensity-dependent damage exceedance probabilities.

Keywords: structural capacity; seismic demand; ground motion selection; fragility; uncertainty and correlation.

1. Introduction

The various sources of aleatory variability (and the correlation of their residuals) associated with ground-motion and structural response predictions cannot be neglected in loss assessment procedures, as demonstrated by several authors (e.g. [1]). Hence, the purpose of this study is to investigate the appropriate treatment of material, geometrical and record-to-record variability in the derivation of fragility models for the earthquake loss estimation of building portfolios. In this context, a number of key ground-motion characteristics such as frequency content and spectral shape; peak ground motion; and duration have been demonstrated to significantly influence predictions of the response of nonlinear systems, which typically renders record-to-record variability the main source of aleatory (i.e. random) variability [2].

Intensity measures (*IMs*) shall ideally embody features of: *efficiency* [2], *sufficiency* [3], *predictability* [4], and *scaling robustness* [5]. However, it is acknowledged in many applications that none of the commonly used intensity measures (*IMs*) are *sufficient* with respect to the distribution of ground motion characteristics – namely, magnitude (*M*), distance (*R*), and *epsilon* (ε) – expected at a given site, as determined by probabilistic seismic hazard analysis *(PSHA*) [6]. Thus, it is clear that the response from nonlinear analysis will be dependent on the suite of selected records, as demonstrated by Haselton *et al*. [7], who assessed the influence of *epsilon* in the collapse fragility of a large number of structures.

As evidenced by Haselton *et al*. [8], a robust mechanism to determine structural response variability for a particular level of seismic action shall be based on a record-selection procedure that incorporates the prediction of both mean and variance of the considered intensity defining parameters. To this end, the Generalized Conditional Intensity Measure (*GCIM*) approach [9] is employed in the selection of natural ground-motion records that are primarily scaled to match increasing levels of spectral ordinates at the mean fundamental period of vibration of the classes of interest - $Sa(T_1)$. According to the latter, conditional distributions of a relevant set of *IMs* are determined by taking into account all the rupture scenarios that influence the seismic hazard at the site of interest – Lisbon, Portugal – by means of the relative contribution of magnitude, distance and ground motion prediction models obtained from disaggregation [10], as formulated in Lin *et al*. [11].

In this study, thousands of nonlinear dynamic analyses were performed within a probabilistic methodology, developed by Silva *et al*. [12], wherein hundreds of reinforced concrete frame models (with distributed plasticity) are simulated in a 2D environment. Through Monte Carlo simulation, the variability in the geometrical and material properties of typical two, five and eight-story pre-code reinforced concrete buildings in mainland Portugal is taken into account. As a result, the minimum number of ground-motion records necessary to achieve robust predictions of response variability is appraised, in order to achieve hazard-consistent and statistically meaningful distributions of structural response, conditioned on different levels of seismic intensity - $Sa(T_1)$. In this framework, matters of *efficiency, sufficiency* and *predictability* are taken into account. Nonlinear response analysis of 100 structural models is performed for each selected ground-motion record; and damage exceedance probabilities are determined for each record, at each level of $Sa(T_1)$. The importance of computing "record-specific" probabilities is highlighted in the context of the loss estimation of building portfolios, which is strongly influenced by the spatial correlation of ground motion intensity parameters. To this end, it is demonstrated that the verified variability of "record-specific" probabilities is conditional on each level of $Sa(T_1)$, and dependent on intensity measures other than $Sa(T_1)$. Thus, these probabilities can be expressed as a function of a conditional intensity measure ($IMi_{|Sa(T_1)}$), which establishes the proposed concept of *conditional fragility function*.

2. Numerical models

Drawing upon the study by Silva *et al*. [12], in which material and geometrical properties of the most representative Portuguese building classes were characterized, the numerical models considered herein represent typical buildings constructed before 1958, the year when the first seismic design provisions were enforced, and are thus defined as *pre-code*.

Dynamic properties are characterized by the mean fundamental periods of vibration of the random generation of assets with varying geometrical and material statistical distributions. These have been found to be 0.26, 0.45 and 0.70 seconds, for the two, five and eight story buildings, respectively. The percentage of reinforcement in the beams and columns is calculated following the pre-code regulations and practices corresponding to the ultimate and serviceability limit states, for each asset, in accordance with the sampled geometrical and material characteristics.

When using a Monte Carlo approach to randomly generate portfolios of buildings, it is important to ensure that convergence in the results is achieved. Accordingly, as demonstrated in a study by Silva *et al.* [13] in which a similar sampling framework was implemented, the use of one hundred assets is necessary to guarantee the statistical significance of the generated distribution of structural capacity. To maintain the computational effort at a reasonable level, each structure is modelled as a single infilled moment frame with three bays. As schematically presented in Fig. 1, for the case of 5 story buildings, each frame was modelled in a 2D environment using the open-source software OpenSees [14], with force-based distributed plasticity beam-column elements. For the sake of synthesis herein, readers are referred to the aforementioned work by Silva *et al.* [12] for details of the numerical considerations adopted with regards to the cross section discretization and integration points of the elements, the material constitutive relationships, P-delta effects, and the infill panel modelling approach.

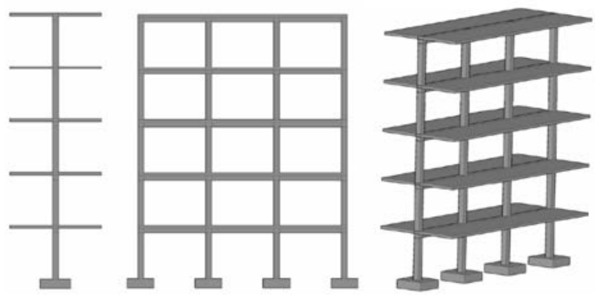

Fig. 1 – Schematic view of the five-story RC frame model: front (left), side (centre) and isometric view (right) without infills, adapted from Silva *et al.* [13].

3. Record selection methodology

In the analytical assessment of building fragility, the record-to-record variability should be robustly modelled given its significant influence on the estimated distribution of structural response. Amongst the available ground motion selection procedures, the Conditional Spectrum (CS), initially proposed by Baker [15] and further developed by Jayaram *et al*. [16], provides a mechanism for estimating both the target mean and variance of spectral ordinates that a set of selected records should match, thus adequately accounting for the record-to-record variability. However, a limitation of the latter approach is that only the characteristics of ground motion represented in terms of spectral ordinates are considered. Thus, the Generalized Conditional Intensity Measure (GCIM) approach proposed by Bradley [9] is adopted herein for record selection, as it allows all the intensity measures identified as necessary to ensure that *efficiency, sufficiency, scaling robustness* and *predictability* are accounted for. A brief summary of the theoretical concepts behind the GCIM is provided below, and its application in the present study is presented further in the following sections.

The fundamental basis of the conditional response spectrum is that spectral accelerations at multiple vibration periods can be assumed to have a multivariate lognormal distribution, and the conditional distribution of spectral acceleration ordinates, for a single earthquake scenario, given the occurrence of a specific value of the spectral acceleration at some period, has a univariate lognormal distribution [16]. In the GCIM, this concept is extended to any ground motion parameter of interest [9]. In other words, the distribution of any *IMi* given an earthquake scenario, or rupture *Rup* (*IMi*|*Rup*), conditioned on the occurrence of a particular level of another intensity parameter (*IMj*), $f_{IMi|Rup,IMj}\left(im_i\middle|rup_k, im_j\right)$, can be assumed to have a lognormal distribution.

Upon the definition of appropriate Ground Motion Prediction Equations (*GMPE*) and correlation models between different intensity measures (*IMi)*, the conditional distribution of each *IMi* given *IMj = imj* is obtained via the total probability theorem as follows:

$$f(IM_i|_{IM_j=im_j}) = \sum_{n=1}^{N_{Rup}} f_{IMi|Rup,IMj}\left(im_i\middle|rup_n, im_j\right) P_{Rup|IMj}(rup_n|im_j) \qquad (1)$$

Where $f_{IMi|IMj}(im_i|im_j)$ is the probability density function (*pdf*) of *IMi* given *IMj=imj*; $f_{IMi|Rup,IMj}(im_i|rup_n, im_j)$ is the *pdf* of *IMi* given *IMj=imj* and *Rup=rup$_n$*; and $P_{Rup|IMj}(rup_n|im_j)$ is the contribution weight of *Rup=rup$_n$*, determined from seismic hazard disaggregation. From the assumption that the vector of all the considered *IMi* (herein referred as *IM*) is characterized by a multivariate lognormal distribution, it follows that for each *IMi*, the function $f_{IMi|Rup,IMj}(im_i|rup_n, im_j)$ has a univariate lognormal distribution, which can be defined by its conditional mean and standard deviation parameters:

$$\mu_{lnIM_i|Rup,IM_j}(rup_n, im_j) = \mu_{lnIM_i|Rup(rup_n)} + \sigma_{lnIM_i|Rup(rup_n)}\rho_{lnIM_i,lnIM_j}\varepsilon_{lnIM_j} \tag{2}$$

$$\sigma_{lnIM_i|Rup,IM_j}(rup_n, im_j) = \sigma_{lnIM_i|Rup}(rup_n)\sqrt{1-\rho^2_{lnIM_i,lnIM_j}} \tag{3}$$

Which are determined as a function of ε_{lnIM_j}, the number of standard deviations, $\sigma_{lnIM_j|Rup}(rup_n)$, by which the logarithm of *IMj=im$_j$* differs from the mean prediction of a particular *GMPE*, $\mu_{lnIM_j|Rup}(rup_n)$, for a given rupture scenario, *Rup=rup$_n$*:

$$\varepsilon_{lnIMj} = \frac{lnIM_j - \mu_{lnIM_j|Rup}(rup_n)}{\sigma_{lnIM_j|Rup}(rup_n)} \tag{4}$$

Accordingly, $\rho_{lnIM_i,lnIM_j}$ corresponds to the correlation of residuals between different intensity parameters, presented in detail in section 3.3.

3.1 Probabilistic seismic hazard and disaggregation

A number of seismic hazard models exist for Portugal, but only one of these has been selected herein for the purposes of demonstrating the methodology to link nonlinear response analysis with *PSHA*. The seismological source model has been taken from the study by Vilanova and Fonseca [18], whilst the selection of the *GMPEs* was performed based on the findings of Vilanova *et al*. [19], in which regional ground motion data from moderate magnitude earthquakes was used to verify the performance of different *GMPEs* in the Iberian region. Subsequently, the models developed by Atkinson and Boore [20] and Akkar and Bommer [21] are considered herein, with 0.70 and 0.30 logic tree weights, respectively.

Typically, causal earthquake magnitude, source-to-site distance and fault properties are considered in the definition of scenarios that contribute to the hazard in a given site, and are established by disaggregation of the *PSHA* [10]. However, following the developments made by Lin *et al*. [11] to the Conditional Spectrum framework, seismic hazard disaggregation should not be limited to Magnitude (*M*) and Distance (*R*), but also consider the influence of different *GMPEs*, in order to ensure the consistency between the target distributions of all considered intensity measures, *IMi*, and the variability of ground motion properties expected at the site of interest (Lisbon; Lat. = 38.373, Lon. = –9.143). Thus, $f(IM_i|_{IM_j=im_j})$ is estimated for each conditioning intensity level according to the contribution of all N_{Rup} scenarios and set of *GMPEs* considered, as described below:

$$f\left(IM_i|_{IM_j=im_j}\right) = \\ = \sum_{m=1}^{N_{GMPE}}\sum_{n=1}^{N_{Rup}} f_{IM_i|Rup,IM_j}(im_i|rup_n, im_j, GMPE_m)P_{Rup,GMPE_m|IMj}(rup_n, GMPE_m|im_j) \tag{5}$$

The OpenQuake-engine [22] which has been used herein for the probabilistic seismic hazard analysis based on rock site conditions (i.e. shear wave velocity in the top 30 m of the soil of 760 m/s) does not currently address 3D disaggregation on *M*, *R* and *GMPE* ; however, due to its open-source nature, it was possible to produce the necessary intermediate results for the computation of $P_{Rup,GMPE_m|IM_j}(rup_n, GMPE_m|im_j)$, as demonstrated below:

$$P_{Rup,GMPE_m|IM_j}(rup_n, GMPE_m|im_j) = \frac{v(IM_j, Rup|GMPE_m).P(GMPE_m)}{v(IMj)} \tag{6}$$

Where $P(GMPE_m)$ stands for the logic-tree weight assigned to $GMPE_m$; $v(IMj, Rup|GMPE_m)$ is the rate corresponding to the conditional probability of *IMj=imj*, using $GMPE_m$, assuming a Poissonian process; and $v(IMj)$ is the rate of occurrence of *IMj=imj*, computed from the corresponding rate of exceedance, as proposed by Bradley [9].

3.2 Record Database

Only three seismic events with significant ground motion were ever recorded in Portugal. For this reason, in order to create a sufficiently large database of candidate records for selection, accelerograms from other regions in the world with similar geological and tectonic characteristics were gathered (e.g. Spain, France, Switzerland, and East United States). The properties of stable continent and active shallow crustal regions, as well as the corresponding faults influencing the seismic hazard were respected to the maximum extent, based on the information provided by Vilanova and Fonseca [18].

3.3 Selected intensity measures

As demonstrated in a study by Sousa *et al*. [23], in which *efficiency* of an extensive set of *IMs* has been evaluated in the context of fragility estimation, there are a number of intensity measures related to duration and number of cycles that do not provide statistically meaningful correlation with the structural response of the building classes considered herein. On the other hand, the intensity measure types that incorporate velocity and spectral shape characteristics systematically provide increased correlations with damage exceedance probabilities. Theoretically, any intensity parameter can be considered in the *GCIM* selection approach. However, the latter assumption hinges on a number of constraints that, in practice, currently limit the number of *IMi* that can be considered:

a) *Predictability* must be ensured, based on the availability of *GMPEs* for predicting marginal mean and standard deviation of the logarithm of each *IMi*;
b) It must be possible to determine the *correlation* between each intensity parameter considered.

The applicability of the selected *GMPEs* to the specific case of mainland Portugal renders spectral acceleration at a range of periods an obvious initial choice for the target *IMi*. Thus, in order to ensure that target distributions computed for *IMi* other than spectral ordinates are consistent with the ground motion properties to be expected at the site of interest, preference is given to *IMi* for which marginal median and logarithmic standard deviation can directly be determined or indirectly be inferred from the same *GMPEs*. Therefore, the vector of intensity measures considered (i.e. *IM*) includes intensity parameters (i.e. *IMi*) of peak ground acceleration (PGA), peak ground velocity (PGV), acceleration spectrum intensity (ASI), Housner intensity (HI) and spectral ordinates within the range of 0.05 to 3.0 seconds, conditioned on the spectral acceleration (*IMj*) at the mean fundamental period of vibration of each class ($Sa(T_1)$). Thus, the probabilistic distribution of the selected *IM* vector conditioned on a given level of $Sa(T_1)$ is designated henceforth as $f(IM|_{Sa(T_1)=a})$; being determined according to the hazard-consistent probabilistic distribution of each *IMi*, given *Sa(T1)=a* - $f(IM_i|_{Sa(T_1)=a})$ - as established in Equation 5, and the correlation models summarized in Table 1 .

Table 1 – Correlation models considered for application of GCIM selection methodology

	$SA(T_i)$	**PGA**	**PGV**	**ASI**	**HI**
$SA(T_i)$	Baker and Jayaram [25]	Baker [26]	Bradley [27]	Bradley [28]	Bradley [28]
PGA	-	-	Bradley [27]	Bradley [28]	Bradley [28]
PGV	-	-	-	Bradley [27]	Bradley [27]
ASI	-	-	-	-	Bradley [28]

4. Fragility assessment

As discussed by Silva *et al.* [12], the use of local criteria to define limit states when generating fragility curves for a population of buildings may not be appropriate. Hence, in the study herein, structural response will be evaluated based on the maximum inter-story drift (*ISD*) and global drift (*GD*), considering four damage states: Slight Damage (*SD*), Moderate Damage (*MD*), Extensive Damage (*ED*) and Collapse (*Col*). In this context, *GD* corresponds to the maximum roof drift ratio, computed as the fraction between maximum roof displacement and building height.

4.1 Limit state criteria

GD limits are determined according to the evaluation of capacity of each frame through a displacement-based adaptive pushover [29]. Similarly to what has been considered by other authors, displacement thresholds at each limit state are defined for each sampled frame without masonry infills (bare frame) according to the following assumptions:

- Slight damage: global drift at 50% of maximum base shear capacity;
- Moderate damage: global drift when 75% of maximum base shear capacity is achieved;
- Extensive damage: global drift at maximum base shear capacity;
- Collapse: global drift when 20% decrease of the base shear capacity is verified, or 75% of the ultimate global drift attained, whichever is achieved first.

The influence of infill panels, which translates to a significant decrease of displacement capacity, is accounted for by applying the reduction factors proposed by Bal *et al.* [30] for each aforementioned limit state.
For what concerns *ISD*, a fixed set of values per limit state are defined based on the evaluation of global damage with increasing inter-story drift from 25 dynamic tests performed in real reinforced concrete moment resisting frames by Rossetto and Elnashai [31]. In order to adapt the six damage states proposed by the latter Authors with the one being considered in this study, *light/slight damage* and *partial collapse/collapse* damage states have been merged, as follows:

- Slight damage: 0.08% maximum inter-story drift;
- Moderate damage: 0.30% maximum inter-story drift;
- Extensive damage: 1.15% maximum inter-story drift;
- Collapse: 2.80% or higher maximum inter-story drift;

4.2 Uncertainty in structural response

In this framework, variability in structural capacity, taken into account through the sampling of $N_F = 100$ frames, has been addressed through a probabilistic approach towards the modelling of material, geometrical and mechanical properties. However, since records are selected and scaled based on target distributions of a set of *IMi* (section 3.3) that have distinct impacts on the spatial distribution of seismic demand, the number of ground motions required to achieve reasonable confidence in the estimated response variability is not known *a priori* [32]. It is recognized in the literature that a large number (greater than thirty) is necessary for the aforementioned purposes (e.g. [8]); nonetheless, an accurate estimate is highly dependent on the parameters used to characterize response, as well as the structural properties itself. This matter is addressed in the following section 4.2.1.

4.2.1 Response variability – minimum number of records

A total of 150 records, selected according to the *GCIM* methodology to match target distributions of *IM* – $f(IM|_{Sa(T_1)=a})$ - is hereby assumed as a sufficiently large sample to provide an accurate evaluation of inter-story drift (*ISD*) and global drift (*GD*) distributions at each level of $Sa(T_1)$ in each sampled frame.
The minimum number of records necessary to achieve identical distributions within a given statistical significance level can thus be determined by comparing the latter with responses resulting from record sets of increasing size, selected to match the same target *IM*. Accordingly, the following methodology is devised, for the purposes of determining a minimum number of records necessary for nonlinear response analysis of 100 synthetically generated structures:

1. Distributions of *ISD* and *GD* resulting from nonlinear dynamic analysis of each of the 100 simulated frames (for 2, 5 and 8 story classes), are determined, using a set of 150 records for each level of $Sa(T_1)$;
2. A similar exercise is repeated for samples of 10 to 140 records (with steps of 10 records). These sets are selected to match the empirical distribution of *IM* derived from the *reference* set of 150 records; which

ensures statistical consistency between distributions of *IM* amongst record samples, as determined by Kolmogorov-Smirnov (KS) goodness-of-fit tests [33] within a 10 % significance level;

3. Empirical probabilistic distributions of *ISD* and *GD* obtained in step 2 are individually compared with the *reference* computed in step 1, for conditioning levels of $Sa(T_1)$ ranging from 0.1g to 1.0g (with 0.1g intervals).

It is assumed that convergence on the mean prediction is achieved prior to convergence on the variance. Therefore, step 3 is performed using the Brown–Forsythe (*BF*) test [34], according to which the hypothesis that two sets of data have equal variance is assessed at the 5% significance level. Fig. 2 illustrates the *BF* test statistic (*p-value*) when comparing variances appraised in step 2 against the *reference* distributions computed in step 1.

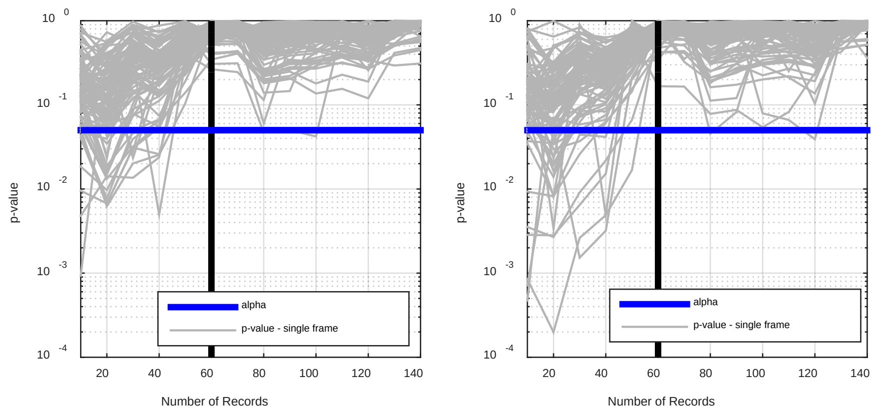

Fig. 2 – BF test statistic (*p-value*) for 100 synthetic 5 story frames; records selected and scaled to a level of $Sa(T_1)$=1.0g. *P-values* higher than 0.05 indicate that the null hypothesis of equal variance cannot be rejected at 5 % significance, for *GD* (left) and *ISD* (right).

This test is preferred over other inference tools such as the F-test for equality of variances [33], which is highly sensitive to departures from normality. As illustrated in Fig. 2, a value of 60 records is considered to provide an adequate compromise between computational effort and statistical significance of results in terms of variance in distribution of Global Drift and Inter-story Drift. Although only the results pertaining to 5 story frames and $Sa(T_1)$ equal to 1.0g are presented, the same conclusion is attained for samples of two and eight story frames, at all considered levels of conditioning seismic intensity parameter, $Sa(T_1)$.

4.2.2 Uncertainty in record-specific probabilities of exceedance

Because 100 frames are analysed for each set of 60 ground-motion records selected for $Sa(T_1)=0.1g$ to *1.0g* (i.e. 60*100 response history analyses per level of $Sa(T_1)$), it is possible to determine damage exceedance probabilities for each ground motion record. In other words, for each level of $Sa(T_1)$, 60 'record-specific' damage exceedance probabilities are determined based on 100 EDP results. As a result, the present framework foresees the characterization of building fragility through probabilistic distributions of damage exceedance probability, denoted as $f\left[Pls_{i\,|Sa(T_1)=a}\right]$. As such, 60 record-specific probabilities of exceedance of *SD*, *MD*, *ED* and *Col*. are estimated according to *ISD* and *GD* criteria for $Sa(T_1)=0.1g$ to *1.0g*, as illustrated in Fig. 3 and Fig. 4.

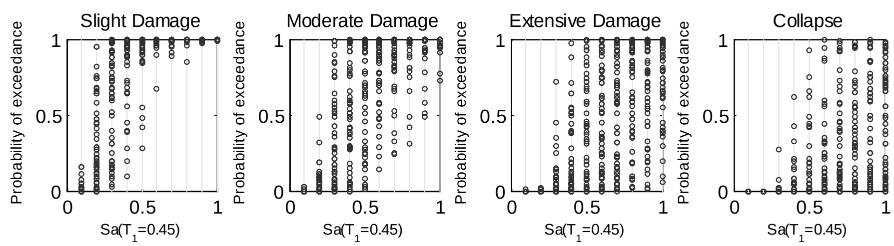

Fig. 3 – Record-specific probabilities of exceedance of *SD*, *MD*, *ED* and *Col*, as a function of *GD* criteruia; for 5-story buildings.

As presented in Fig. 4, the aforementioned probabilities are considered as realizations of random variables, based on which it is possible to determine the associated empirical density function, for each level of $Sa(T_1)$ and damage state. Accordingly, the approximation of a parametric function is evaluated through Kolmogorov-Smirnov (KS) goodness-of-fit tests [33] tests used to assess the null hypothesis that the underlying distributions follow a *Beta* probabilistic model [35].

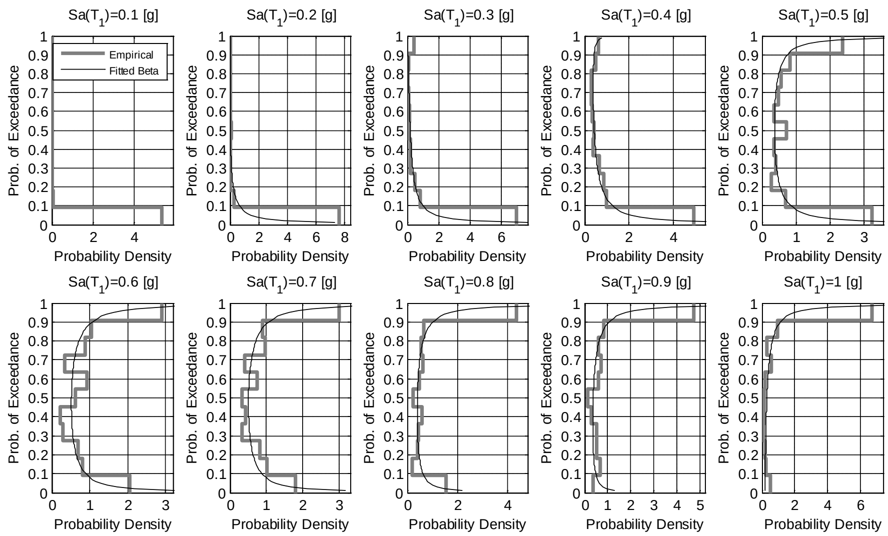

Fig. 4 – Empirical probability density - $f[Pls_{i\,|Sa(T_1)=a}]$ - of damage exceedance probability of *Extensive Damage* and corresponding fitted *Beta* models; damage criteria of *ISD* for 5-story frames.

A visual inspection of the fit between theoretical and empirical distributions illustrated in Fig. 4 highlights the capability of the considered model to take into account variations of probability density in the interval]0.0, 1.0[across different levels of $Sa(T_1)$. The aforementioned null hypothesis is thus verified according to KS tests performed on empirical and theoretical cumulative distribution functions, and cannot be rejected at a 10% significance level. However, it should be highlighted that whenever the *Beta* model is used for the purposes of earthquake loss estimation, appropriate attention should be given to the fact that exceedance probability values of 0.0 and 1.0 cannot be sampled from the latter.

For the sake of synthesis, only the results pertaining to 5-story frames and damage state of *Extensive Damage* evaluated in terms of *GD* criteria are illustrated in Fig. 4. Nonetheless, similar findings regarding the applicability of the selected theoretical model were attained for all structural classes and damage states.

4.2.3 Why determine distributions of record-specific probabilities of exceedance?

In the context of performance-based engineering, it is widely accepted that, in order to appropriately provide a link between seismic hazard and structural response, an “optimal” intensity measure - $Sa(T_1)$ in the present case - must embody features of *efficiency [2]*, *sufficiency* [3], *predictability* [4] and *scaling robustness* [5]. Moreover, provided that *sufficiency*, *predictability* and *scaling robustness* requirements are met, *efficiency* matters are related with the number of analyses necessary for the estimation of a satisfactory approximation to the “true” value of exceedance probability within a specified standard error limit; designated herein as $\tilde{P}ls_{i\,|Sa(T_1)=a}$.

In the present case, the minimum number of sampled frames and ground motion records required for a statistically significant characterization of structural response has been determined. However, distributions of *EDP* and corresponding damage exceedance probabilities are estimated for each ground motion record in each level of $Sa(T_1)$. One might argue that this is an unnecessary step, because $\tilde{P}ls_{i\,|Sa(T_1)=a}$ can simply be obtained from the distribution of 6000 *EDP* values (60 ground motion records *x* 100 frames) for each level of $Sa(T_1)$*;* as illustrated in Fig. 5, in which $\tilde{P}ls_{i\,|Sa(T_1)=a}$ is plotted against the results previously presented in Fig. 3.

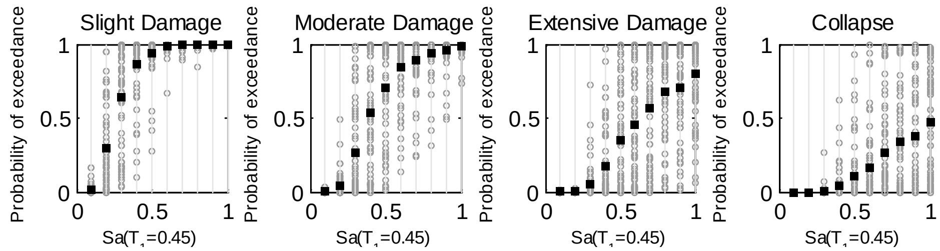

Fig. 5 – Record-specific probabilities of exceedance of *SD, MD, ED* and *Col*, as a function of *GD* criteria; and corresponding $\tilde{P}ls_{i_{|Sa(T_1)=a}}$ (illustrated by the black squares), for 5-story buildings.

However, it is argued by the authors that considering $\tilde{P}ls_{i_{|Sa(T_1)=a}}$ rather than $f\left[Pls_{i_{|Sa(T_1)=a}}\right]$ leads to a misrepresentation of the impact of record-to-record variability in the appraised damage exceedance probabilities and, consequently, in the results of seismic loss estimation.

In order to demonstrate the aforementioned statement, a simple example is presented: the building damage of 2, 5 and 8-story buildings is associated simply with the *Collapse* damage state, with a corresponding *damage ratio*, *DR*, (ratio between the attained loss and the total replacement value of the asset) of 1.0. Strictly speaking, for the purpose of this exercise, the distribution of probabilities of collapse conditioned on $Sa(T_1)=0.5g$ ($f\left[PCol._{|Sa(T_1)=0.5g}\right]$) is equal to the distribution of damage ratios conditional on $Sa(T_1)=0.5g$. Thus, when considering a hypothetical portfolio of 100 buildings of the same structural class located at 100 different sites subjected to the same value of $Sa(T_1)$ *(0.5g* for the purposes of this exercise), the mean (μ_{DR}) and variance (σ^2_{DR}) of the final distribution of aggregated *DR* can be computed according to Equation 7 and Equation 8, respectively:

$$\mu_{DR} = \sum_{k=1}^{100} \mu_{f\left[PCol._{|Sa(T_1)=0.5g}\right]} \tag{7}$$

$$\sigma^2{}_{DR} = \sum_{m=1}^{100} \sum_{n=1}^{100} \rho_{m,n} \cdot \sigma^2{}_{f\left[PCol._{|Sa(T_1)=0.5g}\right]} \tag{8}$$

Where $\mu_{f\left[PCol._{|Sa(T_1)=0.5g}\right]}$ and $\sigma^2{}_{f\left[PCol._{|Sa(T_1)=0.5g}\right]}$ are respectively the mean and variance of $f\left[PCol._{|Sa(T_1)=0.5g}\right]$, which is considered similar in all the *k* locations; and $\rho_{m,n}$ is the spatial correlation coefficient between "record-specific" probabilities at two *m,n* locations.

The characterization of $\rho_{m,n}$ is further addressed in this manuscript, in section 4.2.4. Nevertheless, it is clear from Equation 8 that it plays a very significant role in the loss estimation of spatially distributed portfolios, as evidenced in Fig. 6, where the empirical distribution of *aggregated loss* computed through numerical simulation of the *Beta* approximation to $f\left[PCol._{|Sa(T_1)=0.5g}\right]$ at each site is illustrated for two extreme cases of zero and full spatial correlation.

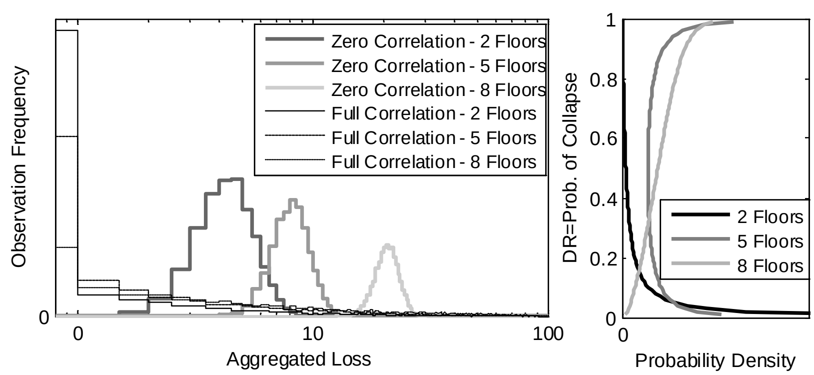

Fig. 6 – Empirical probabilistic distributions of *aggregated loss* computed for a hypothetical portfolio of 100 spatially distributed buildings subjected to $Sa(T_1)=0.5g$, with zero and full correlation between $f\left[PCol._{|Sa(T_1)=0.5g}\right]$ at each of the 100 sites; Limit state criteria of *GD* (left). Beta approximation to $f\left[PCol._{|Sa(T_1)=0.5g}\right]$ (right).

4.2.4 Correlation between damage exceedance probability

The previous section highlights the importance of characterizing building fragility through probabilistic distributions of damage exceedance probability per level of *Sa(T$_1$)*. However, two important questions shall be addressed:

a) Is there a physical meaning underlying the assumption of spatial correlation between $f\left[Pls_{i\,|Sa(T_1)=a}\right]$ (i.e. between "record-specific" damage exceedance probabilities) at different sites?

b) How can such correlation be adequately taken into account, in a hazard-consistent manner?

Regarding a) and b) above, two important aspects shall be evidenced. Firstly, the damage exceedance probability distributions presented in this framework arise from the computation of "record-specific" probabilities of exceedance for each level of *Sa(T$_1$)*. In this context, it is verified that the scatter depicted in Fig. 3 for each level of *Sa(T$_1$)* is the result of record-to-record variability. In other words, for a given level of *Sa(T$_1$)*, the variability in "record-specific" probabilities relates to the variation of a secondary (conditional) intensity measure – $IMi|_{Sa(T_1)=a}$.

When assigning the record-specific values of each $IMi|_{Sa(T_1)=a}$ to the corresponding record-specific probabilities, the more *efficient IMi* can be selected as the one for which the correlation with damage exceedance probabilities of *SD, MD, ED* and *Col* is higher. Furthermore, it is demonstrated in Fig. 7 that, for such $IMi|_{Sa(T_1)=a}$, regression analysis can be performed in order to fit a cumulative lognormal function to the scatter of *IMi*-dependent damage exceedance probabilities. Given its conditional nature, such curves are hereby designated as *Conditional Fragility Functions*, providing a parametric relationship between $IMi|_{Sa(T_1)=a}$ and damage exceedance probabilities when records are selected and scaled for *Sa(T$_1$)=a*.

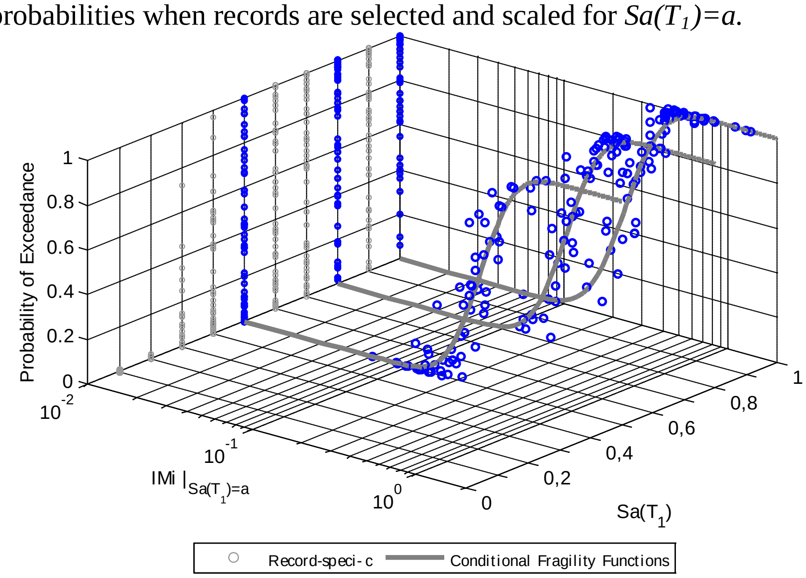

Fig. 7 – Record-specific probabilities of exceedance of *ED*, as a function of *GD* criteria, and corresponding *conditional fragility functions* for the cases of *Sa(T$_1$)*=0.5g, 0.8g and 1.0g; for 5-story buildings

It has been established that $f\left[Pls_{i\,|Sa(T_1)=a}\right]$ can be depicted as a function of a secondary intensity measure conditioned on *Sa(T$_1$)=a*, i.e. $IMi|_{Sa(T_1)=a}$ (Fig. 7). Thus, the second important aspect to be highlighted is the fact that, if the spatial correlation between $IMi|_{Sa(T_1)=a}$ at different sites subjected to identical *Sa(T$_1$)=a* can be determined, then $\rho_{m,n}$ (the spatial correlation between "record-specific" probabilities) has in fact a physical meaning. Since, as established by the *conditional fragility functions*, $f\left[Pls_{i\,|Sa(T_1)=a}\right]$ is a function of $IMi|_{Sa(T_1)=a}$, then "record-specific" damage exceedance probabilities can be assumed as random variables whose uncertainty relates to the record-to-record variability expressed by $IMi|_{Sa(T_1)=a}$. As a result, the spatial correlation between damage exceedance probabilities at different sites subjected to *Sa(T$_1$)=a* is a function of the correlation between the values of $IMi|_{Sa(T_1)=a}$ at those same sites. In this context, the appropriate definition of the correlation between values of $IMi|_{Sa(T_1)=a}$ at different sites subjected to $Sa(T_1)=a$ and the impact of its consideration in a loss estimation procedure is the subject of further research.

5. Conclusions

This papers presents a framework according to which multiple ground motion intensity measures are included in the characterization of building fragility through probabilistic distributions of damage exceedance probability for each level of $Sa(T_1)$. Variability of structural capacity and seismic demand have been considered in an analytical exercise where statistically significant distributions of response have been determined. Moreover, it is demonstrated that even in the case where the number of performed analyses are sufficient to ensure statistically significant distributions of structural response, non-negligible errors can be attained in estimation of damage exceedance probabilities if such computation is not performed in a statistically consistent manner. These errors have furthermore been verified to be dependent on the way the definition of response limit states is performed.
The relevance of the presented novel approach has been demonstrated within the context of loss estimation of building portfolios, where the spatial correlation of ground motion residuals plays a significant role. To this end, the importance of the introduced *conditional fragility functions* is illustrated by demonstrating its capability of consistently take into account record-to-record variability in the evaluation of fragility; while establishing the means by which spatial correlation between damage exceedance probability distributions can be taken into account.

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