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The distribution of landslides of Ludian Ms6.5 Earthquake based on M5' model tree method

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Abstract

On Aug.3, 2014, an earthquake with Magnitude M_s6.5 occurred in Ludian county of Yunnan Province, China. The Ludian M_s6.5 Earthquake resulted in 617 deaths, 112 people missing, and also caused a large number of landslides and a large barrier lake. After the earthquake, a team with an Unmanned Aerial Vehicle (UAV) Disaster Acquisition System was sent to the site and captured the images of the worst hit disaster area. The images are of 0.3 m resolution and cover the entire extreme disaster area so all the landslides can be easily identified in the image. The earthquake-induced landslides which are larger than 5 meters in length, are picked out by comparing UAV imagery with the satellite image before the earthquake.

Combining the high-resolution aerial images with basic geological data of the aerial area, we have considered structural strength of rocks, slope, vegetation condition and fault distance as the main factors affecting the distribution of landslides induced by earthquake. The results show that the landslides are mainly affected by fault distance, slope and structural strength of rocks. The density of landslides is reduces with increased fault distance. Earthquake-triggered landslides form easily at the structural fracture zone and on slopes within 38°-50°.

A piecewise linear model which is fitted by M5' model tree algorithm has four explicit piecewise linear index including the density of landslides, fault distance, structural strength of rocks, slope and vegetation condition. The model approaches the nonlinear relations of earthquake-triggered landslide distribution with those factors. The correlation coefficient between the model and the actual distribution of landslides density is 0.88, and it denotes that landslides density can be determined by the factor referring to the model. Therefore the model might be used to predict the distribution of earthquake-induced landslides.

Keywords: earthquake-triggered landslide; Ludian M_s6.5 earthquake; M5' model tree algorithm; image from UAV

1. Introduction

Landslides trigged by an earthquake are hazardous. There are many studies investigating the relationship between earthquake-trigged landslides and control factors such as peak ground acceleration, the focal depth, fault distance, slope, rock mass structural strength, vegetation condition, and so on [1-5], especially after Wenchuan M_s8.0 Earthquake.

On Aug.3, 2014, an earthquake（27.1°N, 103.3°E, depth 12 km, M_s6.5）occurred in Ludian County of Yunnan Province, China, known as the Ludian M_s6.5 earthquake. The Ludian M_s6.5 Earthquake resulted in 617 deaths, 112 people missing, and also caused a large number of landslides and a large barrier lake. After the earthquake, a team with an Unmanned Aerial Vehicle (UAV) Disaster Acquisition System [6] was sent to the site and captured the images of the worst hit disaster area. The images have a resolution of 0.3m and cover almost the all worst hit disaster area (Fig. 1), capturing the full extent of landslides (Fig. 2). Satellite imagery from Google-Earth is available for the earthquake area in March, 2014, several months before the earthquake. Comparison of the satellite image and the UAV imagery, allows the old landslides and the earthquake-triggered landslides to be distinguished. This is provides the opportunity to study the factors associated with earthquake-triggered landslides in this region.

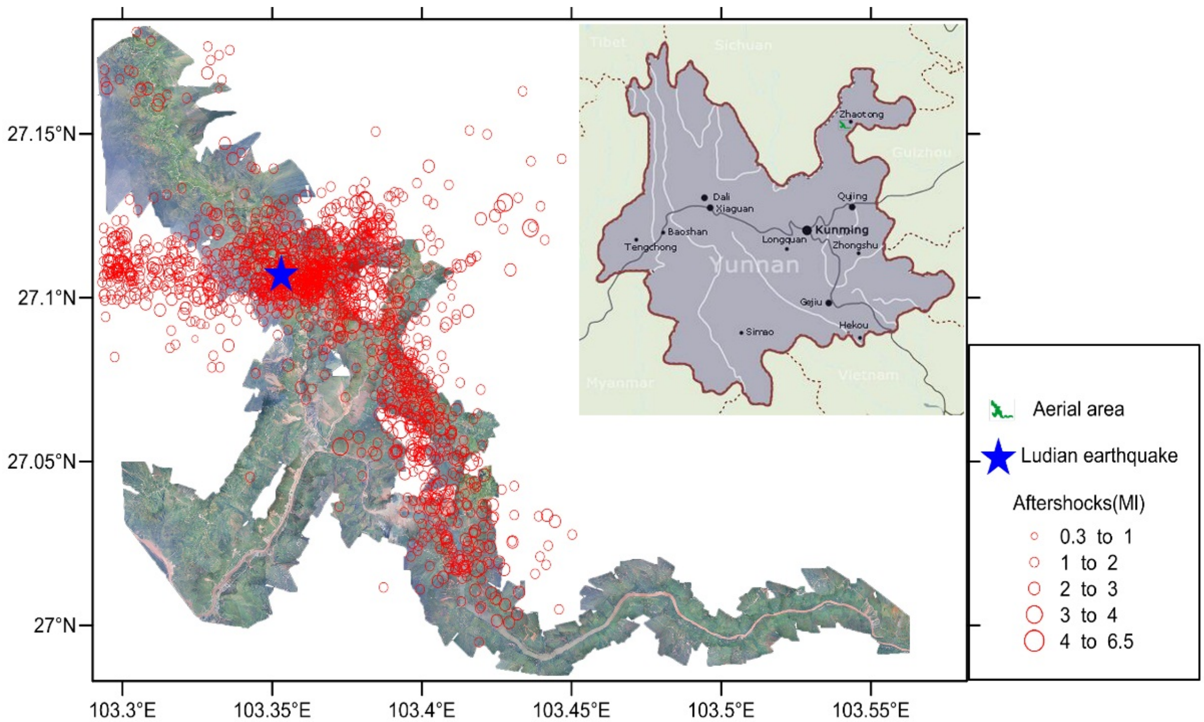


Fig.1 Coverarage of aerial imagery by UAV, the epicenter of the Ludian earthquake and aftershocks (from [7]). Inset shows the survey site within Yunnan Proveinc.

2. Data and process

Many factors control whether a landslide occurs. Based on the past published research [5], the following control variables are considered: distance to fault, structural strength of rock and soil mass, slope, vegetation condition.

2.1 The image of UAV and data processing

After the Ludian M_s6.5 earthquake, the UAV flew 8 surveys, covering the worst hit disaster area[8]. Four thousand, five hundred and sixty photographs were obtained with the minimun resolution is about 0.3 m. Individual images were merged to a single georeferenced image mosaice covering the entire survey area using the software Pix4D. The mosaice image has the resolution of 0.3 m at the main observation area in addition to the edge area (Fig. 2), and covers an area of 298km^2.

The example imagery Fig.2 shows that the landslides are easily distinguished. Earthquake triggered landslides identified in imagery varied in size between several kilometers and several meters in length: only those earthquake-triggered landslides with length larger than 5m are picked out through comparing with the satellite image of Google-Earth for the following statistical analysis. Ignoring the marginal zone, 287 earthquake-triggered landslides were identified and their geographical positions are precisely mapped using the mosaic UAV image.

For each identified landslide, extending 1 km to the surrounding, a square with 4 km^2 is constituted. The number of landslides in the square, which are of more than 5 meters in length, is recorded as the seismic landslide density of the square. For the 287 earthquake-triggered landslides, 287 sample squares are formed and their landslide densities are identified.

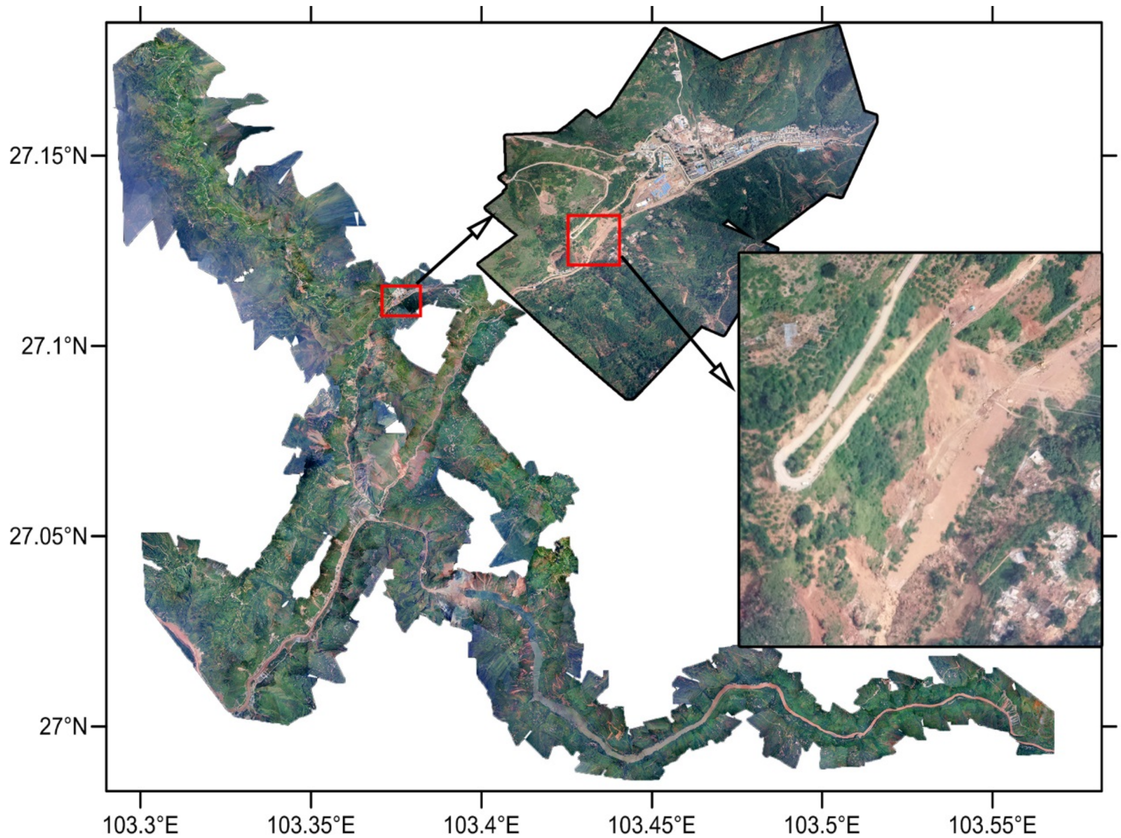
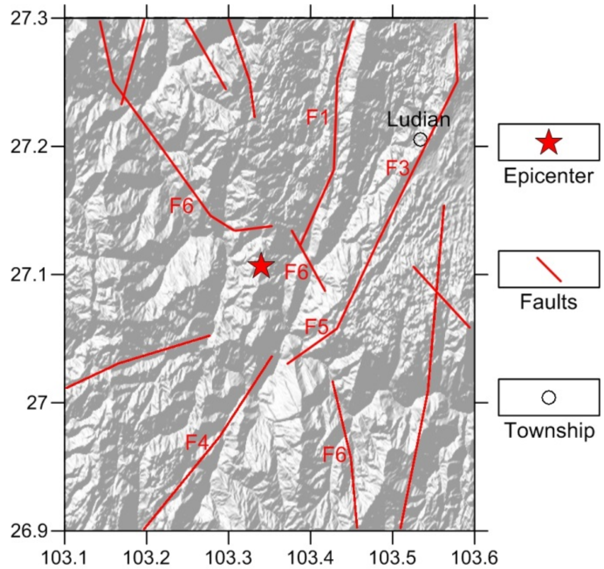


Fig. 2 The mosaic image obtained by UAV. Inset contains an example an example landslide

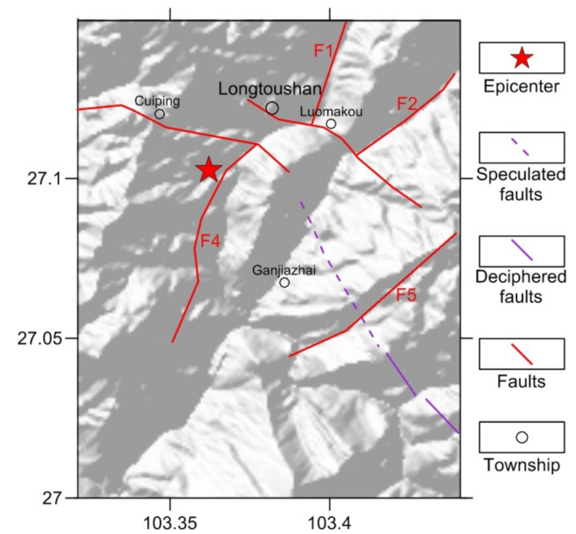
2.2 The geological structure and Classification of structure of rock mass

The Ludian M_s6.5 earthquake occurred on the famous Zhaotong-Ludian fault. The earthquake occured in an area containing NE fault and folds (Fig. 3, Table 1). The major tectonic structures are orientated NE but there are minor NWW faults which cut the NE faults and folds. The Ludian M_S6.5 earthquake originated on the NNW-trending steep left-slip fault that stretches along Baogunao-Xiaohe[9](see Fig.3 (a)), and also the NWW fault because the aftershocks distribution has two directions: NNW and EW (Fig. 1) [9-11]. Also the rupture process of the source shows a complex conjugated ruptured earthquake[12].

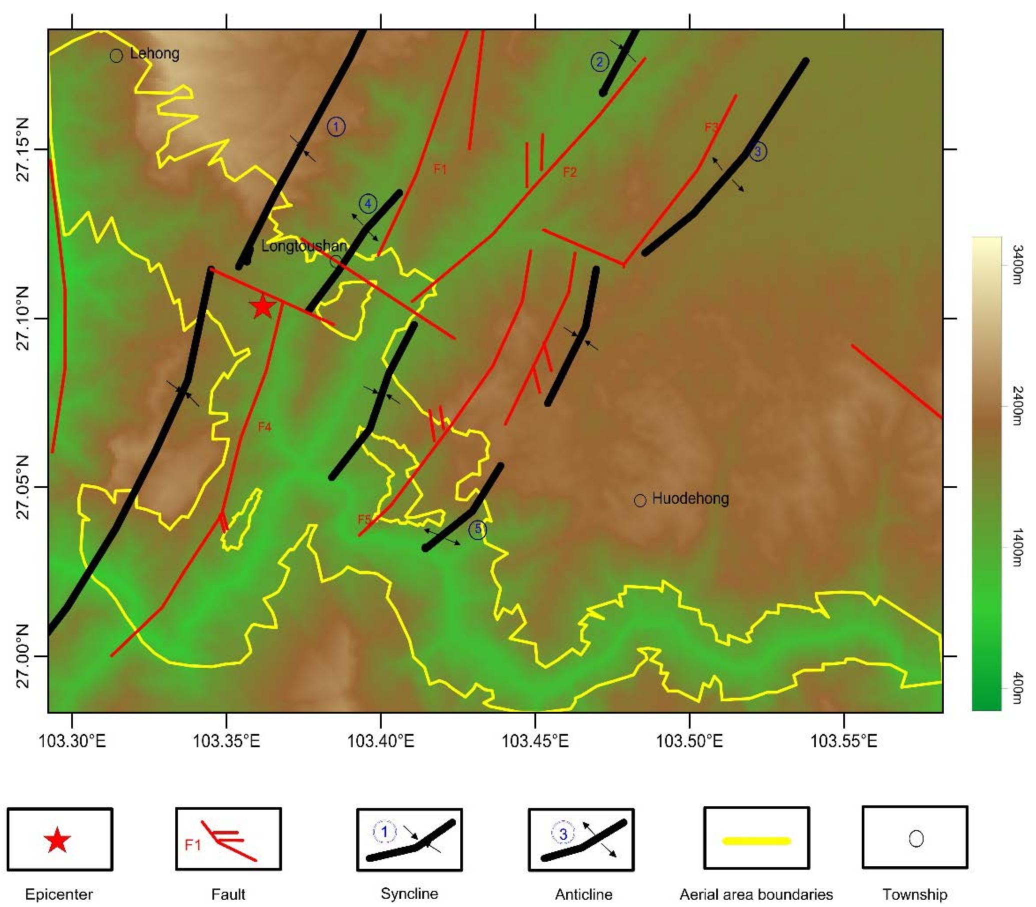
Around this area, there have been 44 earthquakes of magnitude M_S > 5.0 in historical records, and three earthquakes with magnitude M_S > 5.0 since 2013 within 100 km. Every earthquake caused relatively serious disasters. Therefore, the rock mass structure is soft and broken in the area [13].



(a) The fault structure modified from Xu X W [9]



(b) The fault structure modified from Li xi [10]



(c) The geological structure modified from Yunnan Geological Survey [11]

Fig. 3 The geological structure of Ludian M_S6.5 earthquake area. Table 1 provides fault and fold names labelled numerically in these figures

In Fig.3, the names of the faults and the folds are listed in Table 1, and fault F2, F4 and F5 is all belong to Zhaotong-Ludian fault. The faults striked in NW and NWW are belong to Baogunao-Xiaohe Fault. Because of the complex of geological structure, there are different versions about the faults and their name, but they are roughly similar. The three fault structures in Fig.3 are widely used. The new speculted fault in (b) is not named yet. As the result, the center line of the aftershock distribution is used as the fault line of the Ludian M_s6.5 earthquake. The distance to fault of a landslide is the distance of the center of landslide to the fault line.

Table 1 – The geological structure and number of aerial area

Fault	Fold
F1 Wulibei fault	① Alukuai Syncline
F2 Saba fault	②Xiaozai syncline
F3 Ludian fault	③Dadichongzi anticline
F4 Loumakou fault	④Loumakou anticline
F5 Wuandaokanfault	⑤Huodehong anticline
F6 Baogunao-Xiaohe Fault	

The rock mass strength is primarily controlled by restricted by the bedding structure and the degree and nature of fragmentation of the rock mass. In this study, the rock mass classified according the characteristics of the bedding structures and fracturing (see Table 2). The distribution of rock mass is from reference[11].

Table 2 – Classification of structure of rock mass

Rock mass	Quantization level
Tectonic-cataclastic structure	1
Block cataclatic structure	2
Stratified soft-interlayered structure	3
Stratified block structure	4
Block corrosion structure	5

2.3 Physiognomy and vegetation coverage

The Ludian M_s6.5 earthquake area is mountainous, with an altitude between 1000 m to 2800 m. There are three rivers in the area, which are Sabar river, Longquan river and Niulanjiang river. Due to river cutting, there are canyons along the rivers. The epicenter of the earthquake is at Longtoushan town where the Longquan river passes by.

Because of the impact of natural conditions and human factors of the reclamation, The earthquake area has sparse vegetative cover. The highest vegetation coverage is only about 25%.

Based on observatons of vegetation cover in the study area, the vegetation conditions were classified into four grades (Table 3). Vegetation cover of more than 45% of the area is rare, and all collapses and landslides occurred on slopes classified in the first three grades of vegetation conditions.

Table 3 – Classification of vegetation cover

Vegetation coverage	<10%	10%-25%	25%-45%	>45%
Quantization level	1	2	3	4

Because of the lack of the information on the structure of the rock mass at 14 landslide sites, they were eliminated and 273 sample areas were retained for analysis.

2.4 Slope

Based on the SRTM(Shuttle Radar Topography Mission) digital terrain data with resolution of the 30 meters in GRD format, the slope information of each landslide slope is calculated. The slope of a point on the surface is defined as the angle between the tangent plane and the horizontal plane over the point, which describes the degree of inclination of the ground surface at the point.

3 Methodology

To analyse the main controls on landslide generation, the M5' model tree method [14-19] was applied. The M5' model tree algorithm was developed by [14] after [15] proposed the idea of the segmented multiple linear regression tree (M5' model tree), which is a more precise first order linear tree model algorithm.

First of all, sample set is built acording to the considered factors such as discribed above section. Then the sample set is divided as the subsets until that the samples in any subset are of the common features or the number of amples within the subset is less than a gaven value. The final subset is called a leef and the father of the leaves is called brach. This process is called splitting. The liner regression model for every subset (brach and leaf is also called node) was built using the least square mehtod. Predicting the samples in the subsets (branch and leaves) by using the themself liner model and calculating the standard deviation of sample set for every branch and leaf, the leaves are eliminated and the father subset (branch) becames a leef if the prediction effect of a leef is worse than the brach. This process is called Pruning. The linear model of each leef subset is smooted to reduce the discontinuity and a new linear model of the leef subset is constructed for the further numerical analysis. The every step algorithm is described as following.

3.1 Splitting

Based on the principle of sample attribute difference, the sample space is split using recursive paritioning until the number of samples at a node is less than a specified quantity, or that the standard deviation reduction of the subsets are less than a set limit conditions and splitting ends. The standard deviation reduction of the subsets can be expressed as:

$$S_{DR} = sd(T) - \sum_i \frac{|T_i|}{|T|} \times sd(T_i) \tag{1}$$

Here T is a total sample set in a node, T_i is the subset i after the T is split into subsets, $sd\,(T_i)$ is the standard deviation of the subset i. $Sd(T)$ is the standard deviation of the sample set T.

This mothod simulates the process of tree growth: the classification of all samples is used to produce an initial tree model composed of nodes, where tree growth ends with the final leef node. The linear regression method is used to establish a multiple regression equation for every node.

3.2 Pruning

In order to improve the efficiency of the model some subtrees were merged and replaced as leaf nodes using pruning ergodic process initial tree model (the process is called pruning). The recursive analysis is a reduced error pruning method. The linear regression method is used to build the multiple linear equations (model) for every node firstly, then the predicted values of the samples are calculated based on the established models in a node. The errors between the prediced values and the actual values were calculted. Within a brach node, the leaves are eliminated when predict error of a brach node is smaller than the predict error of its leef nodes. The reduced error in the pruning can be expressed as:

$$E_R = |N|R_{MSE} - |N_l|R_{MSEl} - |N_r|R_{MSEr} \tag{2}$$

In formula (2), R_{MSE} is the root mean square error of prediction equation of a node (including all samples under the subtree and leaf), and R_{MSEl}、 R_{MESEr} is respectively the root mean square error of the prediction of left and right node (or leaf). The subtree will be retained when E_R is positive, otherwise the node will be transformed into a leaf node.

3.3 Smoothing

The recursive process of pruning optimized initial model to its simplest possible structure. But this may cause a certain degree of discontinuity between the linear models of the adjacent leaf nodes. The smoothing model tree is that the two multivariate linear equation about the sub node and its parent node for each node are combined for a new linear equation by formula (3), which effectively reduces the negative effect brought by the pruning process [18].

$$f_n = \frac{nf_c + kf_p}{n+k} \tag{3}$$

Here f_p is the fitting equation of parent node, f_c is the fitting equation of sub node, n is the sample number to reach leaf nodes, k is a constant value (usually 15), f_n is the merged equations. When the new equations of sub node has been used and the R_{MSE} variation is less than a certain threshold, the linear equation of sub node will be replaced, otherwise the equation from the smooth processing will not be used.

4 Training and test

4.1 Regression analysis

The total sample set is divided into a training sample set and a test sample set. The training sample set is used to create a quantitative model, and the test sample set is used to evaluate the accuracy of the quantitative model. According to the values of the dependent variables, the total sample set is divided into 6 leef nodes using the method of stratified random sampling. Two hundred and eighteen 218 raw samples (about 80%) were selected for the training sample set, and the remaining samples were used as the test sample set (Table 4), according to the proportion of the total sample set in each node.

Table 4 – Statistical characteristic of original samples, training samples and test samples

		Landslide density (num./4km^2)	Fault distance （km）	Strength of rock mass	Vegetation coverage	Slope (rad)
Total Sample Set	Min.	1	0.0	1	1	0.029
	Max.	34	7.5	5	3	1.498
	Average	13.293	3.05	2.747	1.121	0.583
	standard deviation	9.176	1.95	1.355	0.369	0.265
Training Sample Set	Min.	1	0.0	1	1	0.066
	Max.	34	7.5	5	3	1.498
	Average	12．45	3.0	2.733	1.122	0.614
	standard deviation	8.744	2.0	1.323	0.376	0.266
Testing Sample	Min.	1	0.0	1	1	0.029
	Max.	34	6.6	5	3	1.037

Set	Average	14.925	3.1	2.774	1.118	0.523
	standard deviation	9.804	1.9	1.423	0.357	0.254

The piecewise linear regression is applied using the M5' model tree algorithm to assess the density and environmental variables controlling the landslide distribution within the study area. A model tree constructrd of 6 leaf nodes is obtained (Fig. 4).

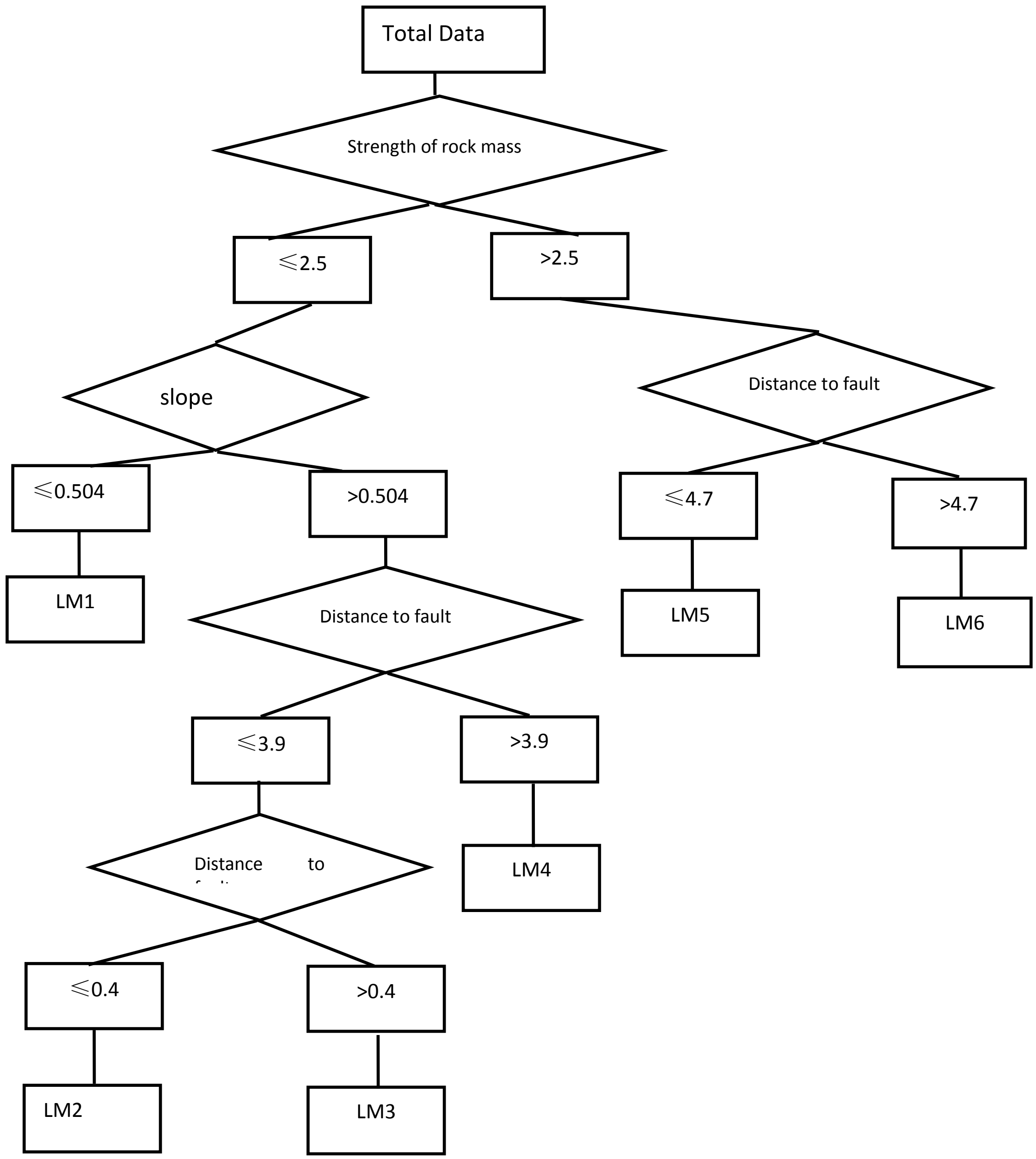


Fig. 4 The model tree with 6 leaf nodes obtained by M5' model tree algorithm

From the structure of the model tree in fig.4, the structure strength of rock and soil mass, slope and the fault distance shows the main reasons for the data splitting process, which means that these three factors controled the distribution of the landslide density in the study area. Based on the three attributes the M5' model tree algorithm divides the relationship between the landslide density and its impact factors into six multivariate linear equations (Fig. 4, Equation (4)-(9)). (F_t is the landslide density, D_f is the fault distance, S_l is slope, R_s is the structural strength of the rock and soil mass, and V_g is vegetation coverage).

When $R_s \leq 2.5$ and $S_l \leq 0.504$, the linear model LM1 is：

$$F_t = -9.2621R_s - 0.9418V_g + 3.1829S_l + 39.1457 \tag{4}$$

When $R_s \leq 2.5$, $S_l \geq 0.504$ and $D_f \leq 0.4$km, the linear model LM2 is：

$$F_t = -4.6007R_s - 0.9469V_g + 3.1829S_l - 0.0939D_f + 29.7451 \tag{5}$$

When $R_s \leq 2.5$, $S_l \geq 0.504$ and 0.4 *km* < $D_f \leq 3.9$ *km*, linear model LM 3 is：

$$F_t = -8.5546R_s - 0.7901V_g + 3.1829S_l - 1.909D_f + 27.4326 \tag{6}$$

When $R_s \leq 2.5$, $S_l \geq 0.504$ and $D_f \geq 3.9$ km, the linear model LM4 is：

$$F_t = -1.3407R_s - 0.72351V_g + 3.1829S_l - 2.1037D_f + 35.5887 \tag{7}$$

When $R_s > 2.5$ and $D_f \leq 4.7$ *km*, the linear model LM5 is：

$$F_t = -2.5218R_s + 0.123S_l - 0.9067D_f + 19.8376 \tag{8}$$

When $R_s > 2.5$ and $D_f > 4.7$ *km*, linear model LM6 is：

$$F_t = 0.7632R_s + 0.123S_l - 1.4485D_f + 24.9641 \tag{9}$$

4.2 Prediction test

The test sample set is used to validate the prediction effects of the equations (4)-(9) which are the piecewise linear models. In the prediction test, 55 test samples and 38 trained samples are used. The predicted values are compared with the actual value (Fig.5). Fig.5 shows intuitively the prediction effect that all prediction values are close to the actual actual values and have a certion errors.

To show the performance of the models, the correlation coefficient and the absolute error of between the predicted and actual values are calculated (Table 5) according to equations (10) and (11) respectively.

$$C_c = \frac{\sum(Actual_i - \overline{Actual})(Predicted_i - Predicted)}{\sqrt{\sum(Actual_i - \overline{Actual})^2 (Predicted_i - Predicted)^2}} \tag{10}$$

The absolute error:

$$M_{ae} = \frac{1}{n}\sum_{i=1}^{n}|Actual_i - Predicted_i| \tag{11}$$

The correlation coefficient C_c indicates the degree of similarity between the actual values and predictive values by regression model: the closer to 1, the smaller the difference between the regression model predictions and the real values. The absolute error M_{ae} is a measure of the difference between the predictive values and the actual values.

Table 5 – Parameters evaluating predictive ability of the regression model

Correlation coefficient（C_c）	0.8756
Absolute error（M_{ae}）	3.2316

9	10.435	2.221
2	6.342	3.257
24	23.096	-0.904
21	16.737	-4.263
34	27.8	-9.588
34	31.3	-6.328
17	24.719	7.719
2	4.172	2.172
23	15.164	-7.836
1.6	3	-1.498
15	23.6	9.421
24	28.3	-1.158
11	14.043	3.043

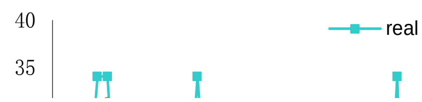


Fig. 5 The prediction of test data set

5 Discussion and results

Using the M5' tree model algorithm, the tree model for lanslide density and its enviromental controls was obtained that is shown in Fig.4 and six formulas of the piecewise linear models were give: equations (4)-(9).

According to the M5' tree model obtained above, the landslide density is strongly clear to the fault distance, rock and soil structure strength, slope and vegetation conditions. The results show that landslide density has a negative correlation with distance to fault, structure strength of rock and soil mass and vegetation conditions, and has a piecewise positive correlation with slope [20] (slope < 50° ,see Fig.6).

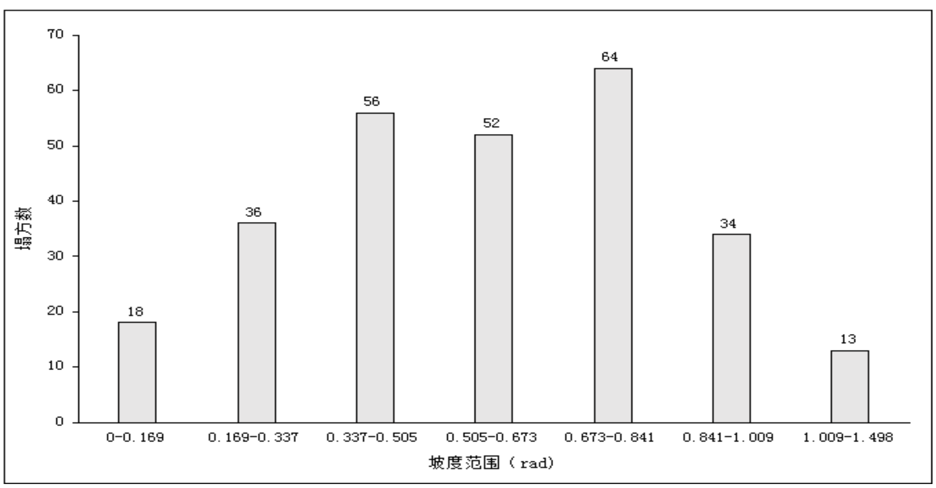


Fig. 6 The distribution of landslides in different scopes of slope

Considering the magnitude of each variable and analyzing each coefficient of variables in six linear piecewise linear regression model, slope, structure strength of rock and soil mass and fault distance plays a master role in landslide density in the study eara. As the vegetation cover in the study area is generally less than 25%, the vegetation value for each landslide point is not very differenct, so the regression model produces a relatively weak relationship between vegetation coverage and the landslide density.

For the Ludian M_S6.5 earthquake, the density of landslides shows little correlation with distance to fault when the distance is close proximity to the fault, < 0.4 km see equation (4). Mabe earthquake energy release of of near fault is in the form of deformation.

The piecewise linear regression model established by the M5' model tree algorithm has a high correlation between the prediced values and the actual values. The regression model produces a good fit between the predicted results of the high landslide density (> 9) in the samples, but the model predictions are not a goog fit when the density of the landslide is small (< 4) in the samples. By analyzing the distribution of the samples, it is found that the sample number of high landslide density is larger than that of low landslide density so that the samples with high landslide density get relatively good prediction. The imbalanced structure of the sample set leads to different prediction performace for different sample space. Maybe another reason is that the aerial area does not cover the entire earthquake disaster area.

A high density of landslides occurs in the area of the structural fragmentation, block fragmentation structure and slope between 20º-50º. The density of landslides decreases rapidly with the distance to fault.

The final M5' tree models contains relatively stable piecewise linear relationships between the density of the landslides and its environment controls from the results of prediction test(see Fig.5 and the Table 5).

The models has the potential to predict earthquake triggered landslide occurrence in this region and the areas with similar environmental characteristics, thereby providing a useful estimate for pre- and post earthquake respone.

The M5'tree model algorithm shows good performace of piecewise linear analysis for the complex sample space in this study.

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References

[1] Meunir P, Hovius N, Haines H J (2008): Topographic site effects and the location of earthquake induced landslides. *Earth and Planetary Science Letters*, **275**, 221-232.

[2] Miles S B, Keefer D K (2009): Evaluation of CAMEL-comprehensive areal model of earthquake-induced landslides. *Engineering Geology*, **104**, 1-15.

[3] Zhang JQ, Fan JR, Yan D (2009): Sensitivity evaluation to the earthquake-induced landslide and collapse--A case study of Beichuan county, Sichuan province. *Journal of Sichuan University: Engineering Science Edition*, **41**(3), 140-145.

[4] Wang H B, Sassa K, Xu WY (2008): Analysis of a spatial distribution of landslides triggered by the 2004 Chuetsu earthquakes of Niigata Prefecture. *Geomorphology*, **101**, 631-642.

[5] Zhang JQ, Su FH, Fan JR (2013): Distribution of landslides and collapses induced by 2013 "4. 20" Lushan earthquake and hazards assessment: a case study of S210 highway. *Journal of Mountain Science*, **31**(5), 616-623.

[6] Xu ZQ, Jiang XD, Zheng Y (2012): The application of UAV on earthquake site. *Recent Developments in World Seismology*, **6**, 204-204.

[7] Fang L, Wu J, Wang W, Lu ZY, Wang CZ, Yang T, Zhong SJ (2014): Relocation of the aftershock sequence of the Ms 6.5 Ludian earthquake and its seismogenic structure. *Seismology and Geology*, **36**(4), 1173-1185.

[8] Yang JS, Xu ZQ, Wang WP, Zheng Y, Peng CY, Chen F, Gao Y, Jiang XD, Hou JS, Yan XD, Li Y (2014): Image set from UAV for Ludian M_S6.5 Earthquake. *Seismological Press*, Beijing.

[9] Xu XW, Jiang GY, Yu GH, Wu XY, Zhang JG, Li X (2014): Discussion on seismogenic fault of the Ludian Ms 6.5 earthquake and its tectonic attribution. *Chinese Journal of Geophysics – Chinese Edition*, **57**(9), 3060-3068.

[10] Li xi, Zhang Jianguo, Xie Yingqing, Miao Qingwen(2014): Ludian Ms 6.5 earthquake surface damage and its relationship with the structure[J]. *Seisimology and Geology,* **36**(4), 1280-1291.

[11] The Second Geological Survey Brigade of Yunnan Geological Survey (1979): Regional Geological Survey Report of Ludian in China. Yunnan.

[12] Zhang Y, Chen YT, Xu LS, Wei X, Jin MP, Zhang S (2015): The 2014 Mw6.1 Ludian, Yunnan, earthquake: A complex conjugated ruptured earthquake. *Chinese Journal of Geophysics – Chinese Edition*, **58**(1), 153-162.

[13] The codification committee of Ludian country annual in Yunnan (1995): Ludian country annual[M], Kunming： Yunnan people's publishing house.

[14] Sattari MT, Pal M, Apaydin H, Ozturk F (2013): M5' model tree application in Daily River flow forecasting in Sohu Stream, Turkey. *Water Resources*, **40**(3), 233-242.

[15] Wang Y, Written IH (1997): Induction of model trees for predicting continuous classes. Poster papers of the 9th European Conference on Machine Learning.

[16] Ross JQ (1992): Learning with continuous classes. *5th Australian Joint Conference on Artificial Intelligence*, Singapore, 343-348.

[17] Clark E T (2010): Refrigerant leak detection system and method. *Patent Application*, 12/943, 626.

[18] Etemad-Shahidi A, Mahjoobi J (2009): Comparison between M5' model tree and neural networks for prediction of significant wave height in Lake Superior. *Ocean Engineering*, **36**(15), 1175-1181.

[19] Zhang JM, Liu D, Wu GZ, Zhang YL (2011): Working condition characteristics identification for extraction unit by using M5' model tree and measured data. *Proceedings of the CSEE*, **31**(23), 21-26.

[20] Tan W, Cai M (2010): Slope generalized reliability theory and practice. *Science press*, Beijing.