16th World Conference on Earthquake, 16WCEE 2017

Santiago Chile, January 9th to 13th 2017

Paper N° 898 (Abstract ID)

Registration Code: S-R1461765417

A STUDY ON PROGRESSIVE COLLAPSE MECHANISM AND UNSEATING FAILURE CONTROL OF CONCRETE GIRDER BRIDGES DURING EARTHQUAKES

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Abstract

The bridge collapse failure is one of the most serious and ubiquity damages during earthquakes. Firstly, three multi-scale models of concrete continuous girder bridge, simply support-continuous girder bridge and completely simply supported girder bridge are established respectively by the finite element software ABAQUS. The nonlinear earthquake responses and collapse processes of three kinds of concrete girder bridges are numerically simulated respectively. Then, the seismic damage evolution laws and progressive collapse mechanisms of three bridges are revealed from different aspects including structural system transformation, bridge pier internal force distribution and girder-pier collision effects. Finally, a new control system for seismic unseating failure of girder bridges is proposed based on the philosophies of energy dissipation, multi-failure and damage reduction. The mechanical behaviors and transformation mode of the new unseating failure control system are investigated. The research results show that the excessive girder-pier relative displacement is the induce factor for bridge unseating and the following pier injuries from the falling span impact action lead to the bridge progressive collapse. Comparatively, for continuous girder bridge, due to the integral continuity of structure, the falling span firstly has great influence on adjacent span and the bridge failure is a whole progressive collapse process. For simply support-continuous girder bridge and simply supported girder bridge, considering the simply supported effect, the influence of falling span firstly on adjacent span is slight and the bridge failure is a partly collapse process. The proposed new control system for bridge unseating failure is effective in preventing span from collapsing and protecting bridge pier from damage through significantly reducing the pier-girder relative displacement with a limited increase of the force experienced by the pier.

Keywords: concrete girder bridges, progressive collapse, unseating failure control, multi-scale, earthquake.

1. Introduction

Concrete girder bridge is a very common type in bridge construction around the world. Destructive earthquakes at home and aboard repeatedly show that concrete girder bridge has great vulnerability and is even collapse failure during earthquakes (see references [1] and [2]). Deeply analyzing the damage accumulation and collapse process of bridges due to seismic excitations, revealing its collapse failure mechanism and proposing a reasonable and effective control system have great theoretical significance and engineering value to improve collapse-resistant capacity of structures and reduce the casualties and property losses in earthquakes.

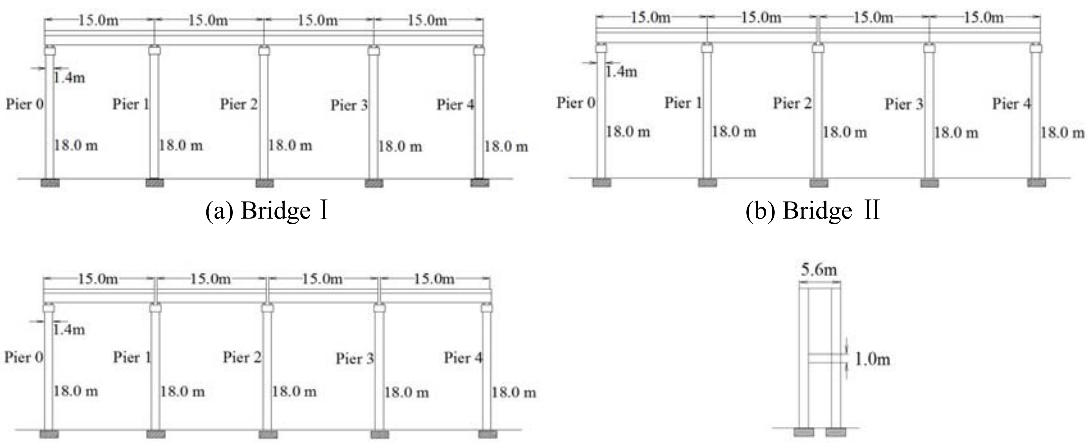
At present, studies of collapse due to earthquakes mainly use discrete element method, meshless method and finite element method. Researches of bridge progressive collapse mainly focus on three aspects: collapse patterns, prevention measures and numerical simulation. Researchers have carried out a series studies on bridge collapse. Miyachi K et al. simulated the collapse process of continuous steel truss bridges and clarified the collapse process, buckling strength, influences of live load distribution and the span ratio on a steel truss bridge (see references [3]). Zhou et al. verified the collapse analysis model proposed by simulating collapse process of CyPress viaduct in America and studied the reasons of bridge collapse (see references [4]). In the study of Bhattacharya S et al., quantitative analysis was carried out for various failure mechanisms that may have contributed to the failure(see references [5]). Julian F D R et al. presented an in-depth analysis to evaluate the efficiency of using cable restrainers connecting isolated and non-isolated spans for preventing unseating of curved steel viaducts[6].

However, there is still a need to develop an appropriate numerical simulation method to accurately reflect and deeply study the damage accumulation collapse process and failure mechanism of bridges with an acceptable computational complexity. Besides, there is few research about multi-level control system to prevent bridges from unseating and even progressive collapse.This paper conducts nonlinear sesimic response of multi-scale microstructure model to reveal the seismic damage evolution laws and progressive collapse mechanisms of three types of concrete girder bridges. And a new multi-level control system for seismic unseating failure of girder bridges is proposed based on the philosophies of energy dissipation, multi-failure and damage reduction. The mechanical behaviors and transformation mode of the new unseating failure control system are investigated.

2. Finite element model of bridge

2.1 Characteristics of three types of multi-span concrete girder bridges

Fig.1 shows the three types of multi-span concrete girder bridge models. Bridge Ⅰ is a four-span continuous girder bridge, bridge Ⅱ is a four-span simply support-continuous girder bridge and bridge Ⅲ is a four-span completely simply support girder bridge respectively. The girders of bridges are plate-type box girders and piers of bridges are double cylinder type piers with collar beam. The detailed bridge parameters are listed in Table 1 and bearing arrangement is listed in Table 2.



(a) Bridge Ⅰ (b) Bridge Ⅱ

(c) Bridge Ⅲ (d) Bridge pier

Fig. 1 – Configuration of three types of multi-span girder bridges

Table 1 – Detailed bridge parameters

Structural component	Length/Height(m)	Width/Radius(m)	Young's modulus(Mpa)
Girders	15.00	6.00	31027
Columns	18.00	0.70	41027

Table 2 – Bearing arrangement

Bridge	Bearing arrangement				
	Pier 0	Pier 1	Pier 2	Pier 3	Pier 4
Ⅰ	sliding	sliding	fixed	sliding	sliding
Ⅱ	sliding	sliding	fixed	sliding	sliding
Ⅲ	sliding	fixed	fixed	fixed	sliding

The sliding bearing is slide type laminated rubber bearing;
The fixed bearing is laminated rubber bearing.

2.2 Multi-scale finite element modeling

The finite element models of three concrete girder bridges are established using the general purpose FE program ABAQUS (ABAQUS 6.12), which contains a comprehensive nonlinear analysis capacity. In order to reflect the damage of bridges and reduce the amount of calculation, multi-scale modeling method is used in this paper. The models use micro element (solid element) to model the parts easy to appear nonlinear deformation and damage such as top and bottom of piers and connection part of pier and collar beam. Other bridge components are modeled using macro element (beam element). The force equilibrium condition is applied to the interface connection of different scale elements (see references [7]). Fig.2 shows the basic multi-scale finite element model of concrete girder bridge.

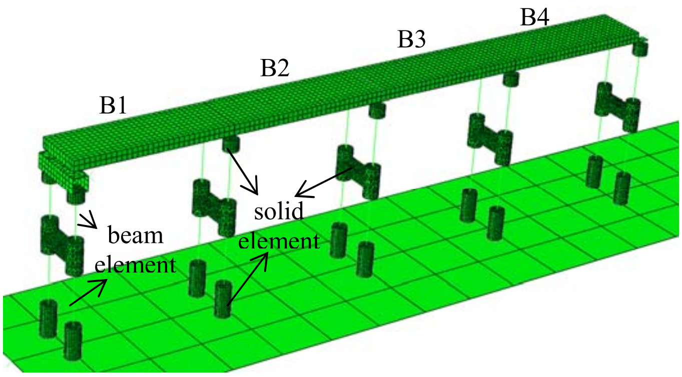
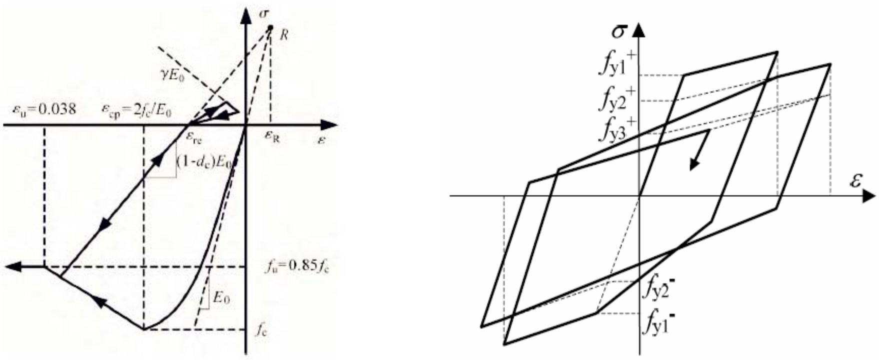


Fig. 2 – Basic multi-scale finite element model of bridge

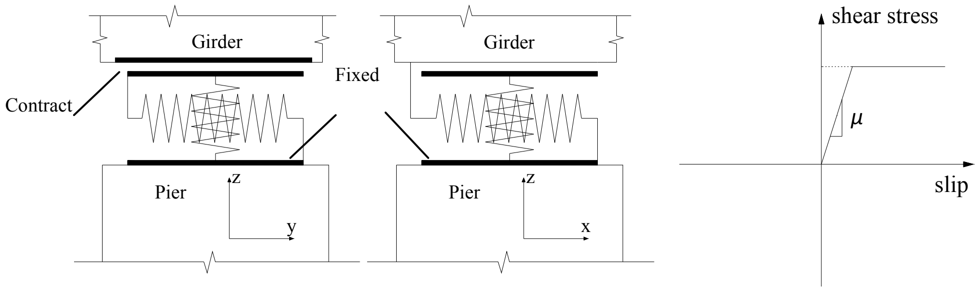
The serial numbers of main girder spans are B1 to B4 respectively as shown in Fig.2. Concrete of superstructure and piers is modeled using plastic damage constitutive model proposed by McKenna F T (see references [8]). Steel bar are modeled using multi-line constitutive model considering the degeneration of the flexural capacity caused by the accumulation damage (see references [9]).The constitutive model of concrete and steel are shown in Fig.3.



(a) concrete (b) steel

Fig. 3 – Material constitutive model

Bearings are modeled using elastic connection element. The model of laminated rubber bearing is shown in Fig.4.



(a) Sliding direction (b) Constraint direction (c) Coulomb friction model

Fig. 4 – Bearing model

The compression deformation stiffness K_V is calculated using equivalent elastic modulus, which can be expressed as:

$$K_V = E_e A_{re} / \sum t_e \tag{1}$$

Where E_e is equivalent elastic modulus considering the effects of triaxial compression; A_{re} is the effective compression area considering shear deformation; $\sum t_e$ is the total thickness of the rubber layer.

The shear stiffness of laminated rubber bearing K_H is directly calculated according to the rubber material parameters, can be expressed as:

$$K_H = G_d A_r / \sum t_e \tag{2}$$

Where G_d is the dynamic shear modulus of laminated rubber bearing, which generally takes $1200\,kN/m^2$; A_r is the shear area of rubber bearing which is calculated by the area of steel plate.

Sliding bearing (slide type laminated rubber bearing) is idealized as coulomb friction model and it is simulated using bilinear connection element. The friction force can be expressed as:

$$F_f = \mu N \tag{3}$$

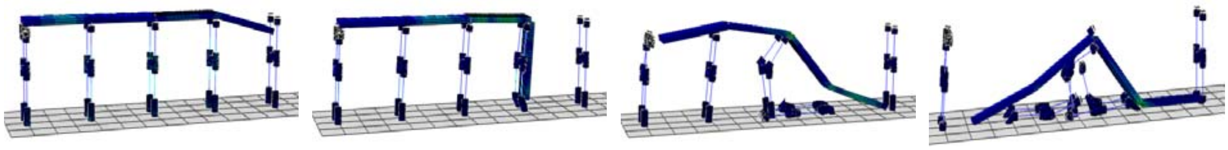
Where μ is the friction coefficient, which takes 0.02; N is the vertical pressure of bearings.

Pounding between girders is modeled using Kelvin collision contact model, combining with automatic search contact algorithm in ABAQUS. Pounding type of bridge structural components is between rigid body impact and collinear impact between rods. This paper takes 0.5 times axial stiffness of main girder. Pounding stiffness between piers and girders takes 10 times stiffness of piers. This model ignores the influence of soil-structure interaction and simulates the ground as a rigid plane, and all bridge piers are fixed on the ground. The seismic responses of three types of bridge models are evaluated using *N-S* component of 1940 El-Centro seismic wave with a peak acceleration of 0.4*g*. This ground motions are applied to the bridge models in the principal orthogonal axes – longitudinal, transverse and vertical.

3. Analysis of collapse process and mechanism of three types of girder bridges

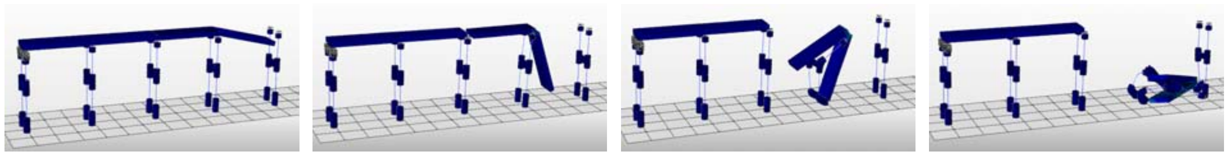
3.1 Progressive collapse process analysis

The numerical simulation of progressive collapse process and structural system transformation of three bridge models are shown in Fig.5–Fig.7 respectively.



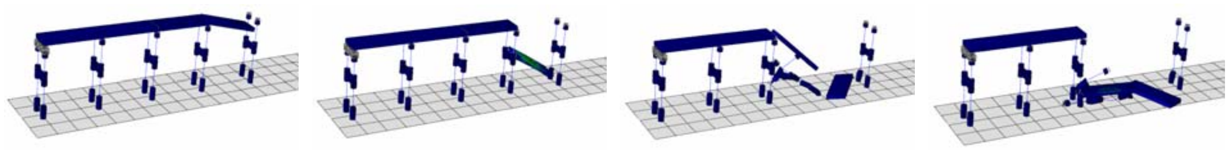
(a) t=0.8s (b) t=2.6s (c) t=4.8s (d) t=6.5s

Fig. 5 –Progressive collapse process of bridge Ⅰ (continuous girder bridge)



(a) t=0.9s (b) t=1.9s (c) t=3.1s (d) t=7.1s

Fig. 6 –Progressive collapse process of bridge Ⅱ (simply support-continuous girder bridge)



(a) t=0.9s (b) t=2.3s (c) t=5.1s (d) t=7.5s

Fig. 7 –Progressive collapse process of bridge Ⅲ (simply support girder bridge)

The progressive collapse of continuous girder bridge continued for about 6.5s, and its structural system transformation process is as follows: Firstly, due to the seismic action, the excessive longitudinal relative displacement occurred between right side of girder B4 and top of pier 4 and girder B4 became a cantilever girder from a continuous beam after unseating, as shown in Fig.5(a). Girder B4 impacted pier 3 after unseating because of large plastic deformation of its left side as shown in Fig.5(b). Large plastic deformation occurred at the bottom of pier 3, and then pier 3 collapsed and impacted pier 2. After that, the whole bridge inclined to the longitudinal direction and girder B1 fell like the situation of girder B4 as shown in Fig.5(c). Then, pier 1 collapsed and impacted pier 2 and unseating of girder B2 happened. Pier 2 collapsed due to suddenly increasing of its load and pounding of pier 1 and pier 3. Finally, the whole bridge collapsed completely as shown in Fig.5(d).

The progressive collapse process of simply support-continuous girder bridge is as follows: Firstly, unseating of girder B4 occurred because of excessive longitudinal relative displacement between it and top of pier 4 as shown in Fig.6(a). Unseating of girder B4 made right part of the bridge incline to the longitudinal direction due to the simply support effect as shown in Fig.6(b). The excessive plastic deformation occurred at the bottom of pier 3 because of unseating of girder B3 and B4, and then pier 3 collapsed. However, the left part of the bridge vibrated with the earthquake and was influenced slightly as shown in Fig.6(c). Finally, the right part of the bridge collapsed completely as shown in Fig.6(d).

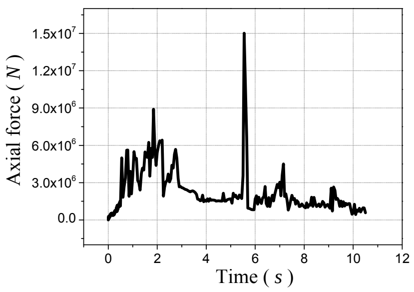
The progressive collapse process of simply support girder bridge is as follows: Firstly, as other two bridges, unseating of girder B4 happened because of excessive longitudinal displacement between it and top of pier 4 as shown in Fig.7(a). The unseating of whole girder B4 occurred because of bridge structural properties and its left side impacted tie beam of pier 3 as shown in Fig.7(b). Pier 3 inclined to the longitudinal direction and collapsed due to pounding of girder B4. The unseating of girder B3 occurred because of large relative displacement between its left side and top of pier 2. The left two spans were influenced slightly under seismic action as shown in Fig.7(c). Finally, pier 3 totally collapsed and the right part of the bridge collapsed completely as shown in Fig.7(d).

3.2 Collapse failure mechanism analysis

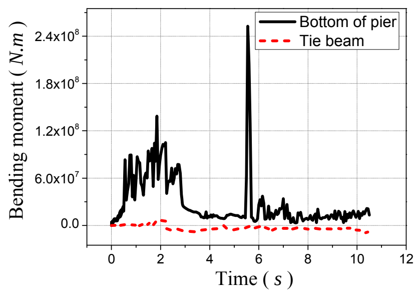
3.2.1 Continuous girder bridge

It can be seen from Fig.3 that collapse of bridge I is a whole transformation process. Unseating is the induce factor for bridge collapse in seismic ground motions and the destruction of bridge piers is the important reason for bridge progressive collapse.

There is basically no damage in bridge girders except plastic hinge damages above bearings. Bridge piers are the component damaged seriously in the progressive collapse process. The change trend of axial force and bending moment of piers is consistent with each other and its maximum value occurs at the time before or after pier destruction. It is consistent with actual situation as shown in Fig.8. Axial force and bending moment of collapsed piers are very large and bending failure occurs at the bottom of piers and tie beams. The bending damage direction of the bottom of piers is different from tie beams, because bridge tilt direction is opposite to the direction of bridge piers impacted as shown in Fig.8(b). Due to the integral continuity of structure, the collapse process of continuous girder bridge is a whole progressive collapse process.



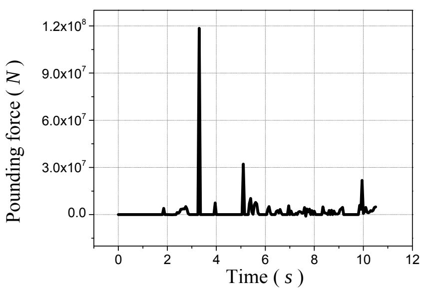
(a) Axial force of bottom of pier 3



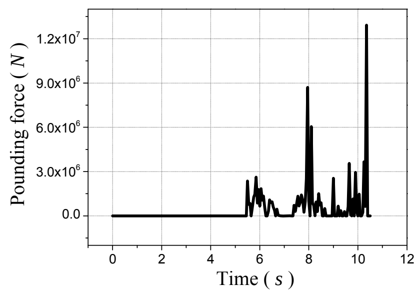
(b) Bending moment of pier 3

Fig. 8 – Internal forces of pier 3

The pounding forces between girders and piers are shown in Fig.9. The peak value of pounding force occurs at time of 3.2s to 3.4s and after 5.0s as shown in Fig.9(a). The former is the pounding force between girder B4 and pier 3 which is larger and lasts longer. The latter is pounding force between girder B4 and ground. The pounding force of girder B1 occurs around 5.5s, it is more frequent but lasts shorter than that of girder B4 as shown in Fig.9(b).



(a) Pounding force of girder 4



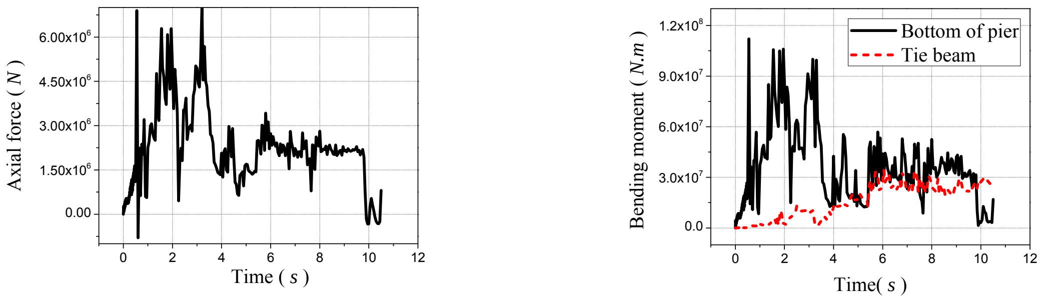
(b) Pounding force of girder 1

Fig. 9 – Pounding force between girders and piers

3.2.2 Simply support-continuous girder bridge

Because of the simply support effect of bridge II, the collapse process of simply support-continuous girder bridge is a partly collapse process.

The damage state of bridge Ⅱ is similar to bridge Ⅰ. Bridge piers of bridge Ⅱ are the component damaged seriously during the collapse process and it is the important reason for collapse of right part of bridge. Axial force and bending moment of bridge piers are also like those of bridge Ⅰ as shown in Fig.10. The bending damage direction of the bottom of piers is also different from tie beams as shown in Fig.10(b). Axial force and bending moment of collapsed pier are very large and bending failure occurs in the bottom of pier 3 and its tie beams.



(a) Axial force of bottom of pier 3 (b) Bending moment of pier 3

Fig. 10 – Internal forces of pier 3

The pounding force between girder and pier is shown in Fig.11. The pounding force of girder B4 has some significant peak values, and the maximum is about 11000*kN*. It can be seen that pounding between girder and pier is relatively severe, initial pounding force is large and pounding time lasts long.

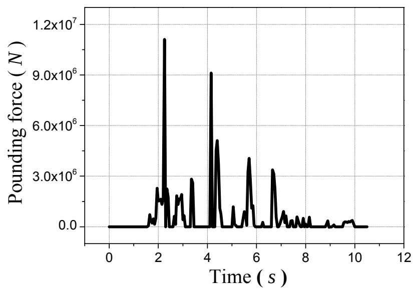
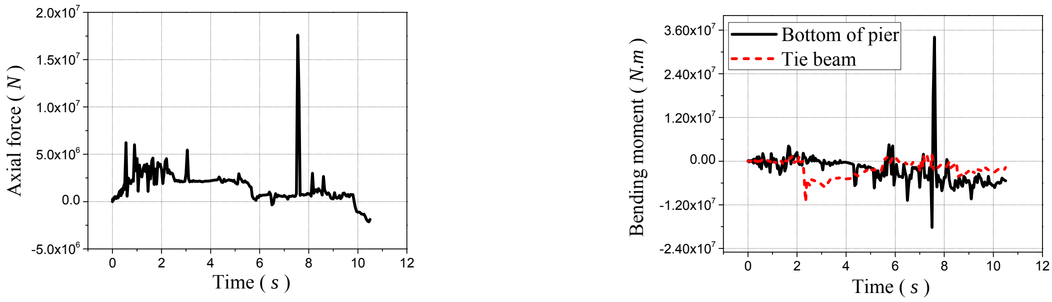


Fig. 11 – Pounding force between girder and pier

3.2.3 Simply support girder bridge

The collapse process of simply support girder bridge is quite different from the other two bridges above. Because bridge Ⅲ is a totally simply support bridge, girder B4 impacts pier 3 after unseating and pier 3 is destroyed. Then, unseating of adjacent girder happens due to collapse of pier 3. The left two spans are influenced slightly. Therefore, the collapse process of bridge Ⅲ is also a partly collapse process.

Plastic hinge damage occurs in bridge girders above bearings. Pier 3 damages seriously because of pounding by whole girder B4 as shown in Fig.12(b). The destruction of pier 3 is the important reason for the collapsing of right part of bridge. The change trend of axial force and bending moment of pier 3 is like the other two bridges as shown in Fig.12. Bending failure occurs in the bottom of pier 3 and its tie beam.



(a)Axial force of bottom of pier 3 (b)Bending moment of pier 3

Fig. 12 – Internal forces of pier 3

The pounding force between girder and pier is shown in Fig.13. The pounding force of girder B4 has some significant peak values, and the maximum value is about 32600*kN* which is larger than that of bridge II. It can be seen that pounding between girder and pier is relatively severe, and initial pounding force is larger than bridge II but pounding time lasts shorter.

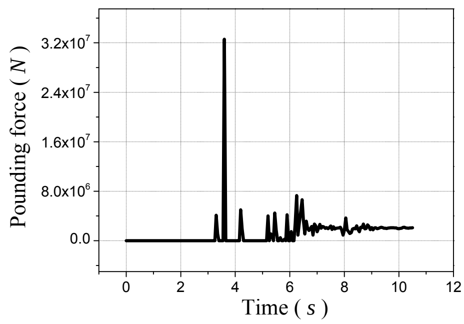


Fig. 13 – Pounding force between girder and pier

4. A new control system of unseating failure

4.1 Establishment of a multi-level unseating control system

Based on the above analysis, excessive relative displacement between pier and girder and pier damage are the key factors of bridge collapse. At the same time, according to both the unseating failure mechanism of concrete girder bridge and the deficiency of existing structural mode of unseating prevention systems, a new type multi-level unseating failure prevention system in this paper is created considering the following three aspects: (1)Through passive energy dissipation mechanism to reduce the structural earthquake responses,realizing seismic energy dissipation design philosophy; (2) According to different earthquake action levels to determine different performance control objectives realizing multi-failure criteria; (3) Through setting “structural fuse” to attain change of multi-level control state and avoid unrepairable damage of important components due to application of restrainer realizing damage reduction philosophy.

On the basis of the above mentioned factors, the new energy dissipation-based multi-level control system for unseating failure prevention can be established as shown in Fig.14.

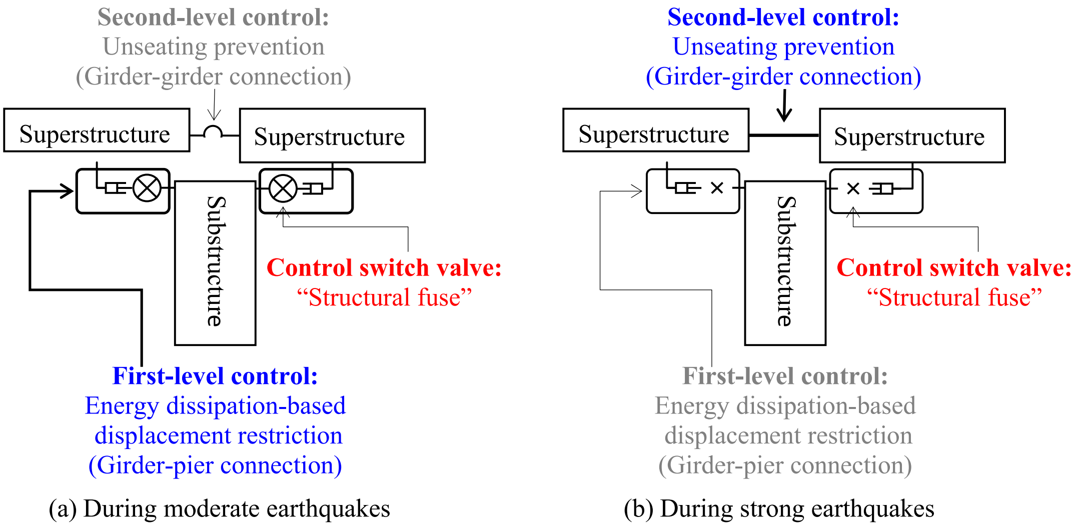


Fig. 14 – Illustration of working mechanism of multi-level control system of unseating failure

In the multi-level unseating prevention system, the first-level control function is energy dissipation-based displacement restriction when the small earthquakes and moderate earthquakes happen, and relative displacement between span and pier of bridges can be reduced by restrainer device between girder and pier. While the threshold value of control switch valve is reached, the unseating prevention system can be transformed automatically to the second-level prevention mode. Thus, the control switch valve is also regarded as a "structural fuse" to avoid unrepairable damage of structure due to excessively large load transferred into pier. The second-level control function is unseating prevention and the span collapse can be prevented by mechanical connection between adjacent girders during strong earthquakes.

4.2 Numerical analysis of mechanical behaviors and transformation mode

The four-span simply support girder bridge (bridge Ⅲ) is analyzed to investigate the working mechanism and the effectiveness of multi-level control system of unseating failure. Six multi-level unseating prevention devices are installed at the expansion joint respectively between the left and right span and transition pier. Four girder-pier connection devices are installed between outside pier and span. Arrangement of unseating prevention devices of bridge Ⅲ is shown in Fig.15.

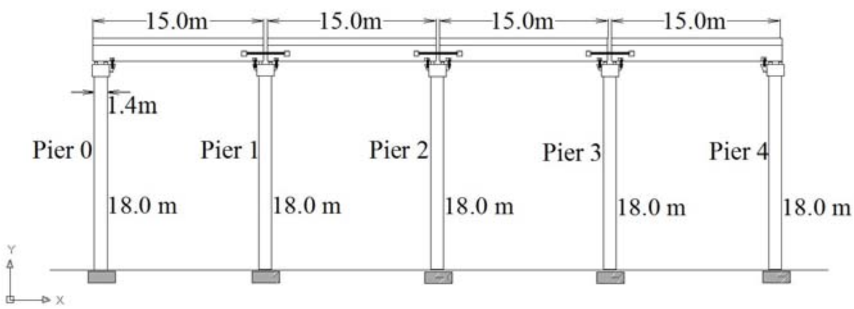


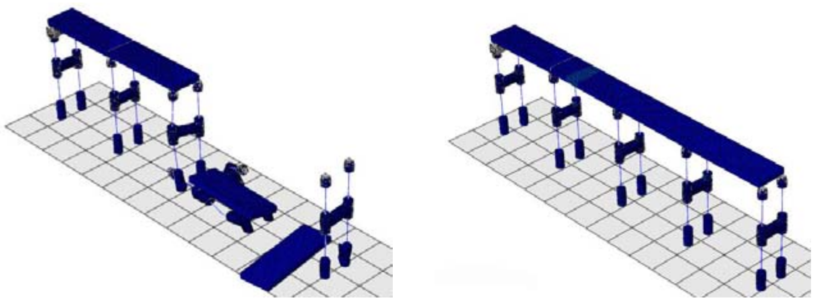
Fig. 15 – Arrangement of unseating prevention devices

The restrainer between adjacent spans is modeled by using "axial" type connector element in ABAQUS. The restrainer between span and pier is modeled by using "axial+align" type connector element in ABAQUS. The earthquake input motion is the *N-S* component of the 1940 El Centro earthquake with a peak acceleration of 0.4*g*. This ground motion is applied to the bridge models in the principal orthogonal axes – longitudinal, transverse and vertical. The viscous damping ratio of structure is assumed to be 5%. The parameter values of finite element model of unseating prevention devices are given in Table 3.

Table 3 – Parameter values of the simplified model

Parameters of unseating prevention devices	Value
Stiffness of restrainer between pier and girder	1.75×10^7 N/m
Strength threshold of restrainer between pier and girder	1.8×10^6 N
Stiffness of restrainer between girders	1.75×10^7 N/m
Damping coefficient	600 N S/m
Gap of girder-pier restrainer	2×10^{-2} m
Gap of girder-girder restrainder	5×10^{-2} m

The responses of bridge due to earthquake ground motions are computed with nonlinear time history analysis method. In order to evaluate the effectiveness of multi-level unseating prevention system, the nonlinear time history analysis is carried out for two analysis cases in this study: Case 1 is a bridge without unseating prevention devices; Case 2 is a bridge with multi-level unseating prevention devices. Because the relative displacement between girder and pier and the seismic force of pier are the main causes affecting the unseating of span and the damage of pier, the response results are respectively given in this paper. The results of different cases under the action of seismic are shown in Fig.16. It can be seen from Fig.16 that unseating of bridge is avoided in case 2.



(a) Case 1 (b) Case 2

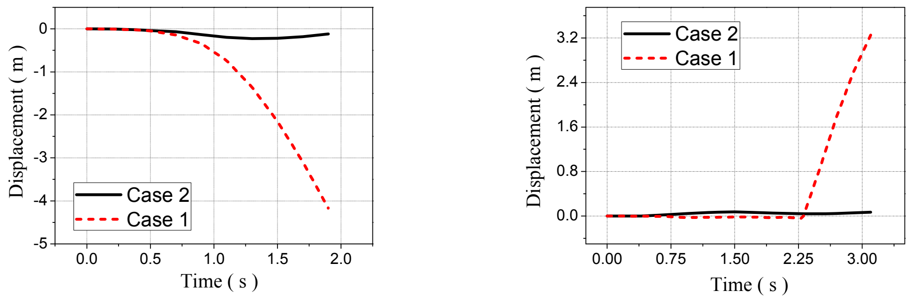
Fig. 16 – Comparison of seismic response of bridges in two cases

The time history responses of the relative longitudinal displacement between girder B3 and B4 and the relative longitudinal displacement between girder B3 and pier 3 are shown in Fig.17.

As can be seen in Fig.17, in the case of bridge without unseating prevention restrainers, the relative longitudinal displacements between superstructures and pier are considerably large, which leads to unseating of girders. Under the action of multi-level unseating failure control system, the peak values of girder-girder and pier-girder longitudinal relative displacement reduce to 2.20×10^2mm and 2.31×10^2mm respectively. The effect of multi-level unseating failure control system is obvious, and the relative longitudinal displacement is controlled well. Therefore, unseating of bridge is avoided in case 2.

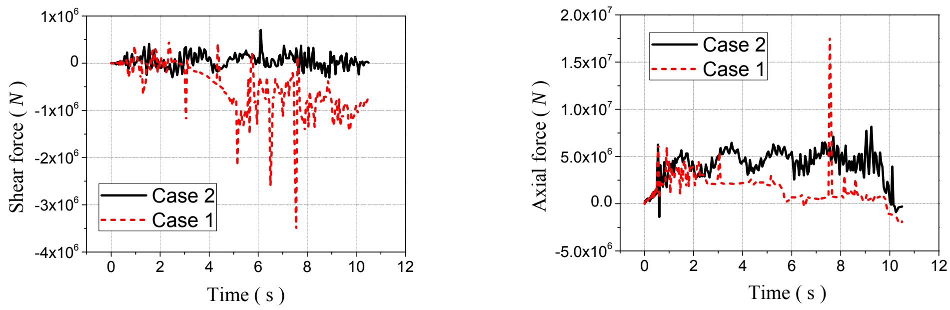
The longitudinal shear force and axial force of pier 3 are shown in Fig.18.

It can be seen from Fig.18 that in case 1, longitudinal shear force of bottom of pier 3 is considerably large because of the effect of bridge unseating and girder pounding. However, under the action of the multi-level unseating failure control system, longitudinal shear force reduced obviously and its peak value is only 704.7*kN*. Because of pier pounding effect, axial force of bottom of pier 3 in case 1 is also large and its peak value exceed bearing capacity of pier. In case 2, axial force is controlled by unseating failure control system, it is smaller than that of case 1. After bridge unseating in case 1, axial force of bottom of pier 3 in case 2 is a little larger than that in case 1 but do not exceed bearing capacity of pier. The peak value of axial force of bottom of pier 3 in case 2 is 5741.2kN and there is no damage in pier3 under the seismic action. Therefore, internal force of bridge piers is controlled well by the multi-level unseating failure prevention restrainers.



(a) Displacement between girders (b) Displacement between girder and pier

Fig. 17 – Longitudinal displacement between spans and pier



(a) Longitudinal shear force of bottom of pier 3 (b) Axial force of bottom of pier 3

Fig. 18 – Internal force of Pier 3

The acting force of unseating failure control system is shown in Fig.19. It can be seen from Fig.16 that first-level control restrainer began to work firstly, and its failure is induced by that acting force of girder-pier restrainer reached strength threshold of the restrainer. Then, second-level control restrainer began to work. The two control levels of unseating failure control system all worked in the earthquake, the acting force of first-level device is obviously larger than second-level device. The unseating failure control system played an important role in concrete girder bridge unseating and reduced its sesimic response.

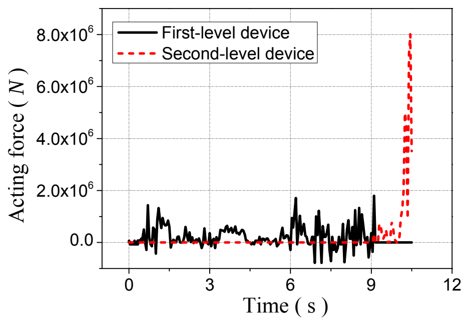


Fig. 19 – Acting force of unseating failure control system

5. Conclusions

In this paper the seismic damage evolution laws and progressive collapse mechanisms of three types of girder bridges are analyzed from different aspects including structural system transformation, bridge pier internal force distribution and girder-pier collision effects. The mechanical behaviors and transformation mode of a new multi-level failure control system are investigated. Several conclusions can be obtained as follows.

(1) The excessive girder-pier relative displacement is the induce factor for bride unseating and the following pier injuries from the falling span impact action lead to the bridge progressive collapse.

(2) For continuous girder bridge, due to the integral continuity of structure, the falling span occurred firstly has great influence on adjacent span and the failure of the bridge is a whole progressive collapse process while the failures of the other two bridges, simply support-continuous girder bridge and simply supported girder bridge, are partly collapse process considering the simply supported effect.

(3) There is no other damage in girders except plastic hinge damage above bearings. Bending moment of the top and bottom of piers and tie beams is large and bending failure occurs easily in these parts; The connection part of tie beams and piers are prone to shear failure.

(4) The application of multi-level unseating failure control system can significantly reduce the maximum relative displacement and internal force of bottom of piers. Therefore, the proposed multi-level control system for unseating failure proves to be effective in preventing span from collapsing and protecting bridge pier from damage.

6. Acknowledgement

The research has been supported by the National Natural Science Foundation of China under Grant No. 51308293, Natural Science Foundation of Jiangsu Province under Grant No. BK20130937 and Natural Science Foundation of Colleges and Universities in Jiangsu Province under Grant No. 13KJB560003.

References

[1] Li H, Lu M, Wen Z, Luo R (2009): aracteristics of Bridge Damages in Wenchuan Earthquake[J].*Journal of Nanjing Tech University* (*Natural Science Edition*), **31**(1):24-29.

[2] Mylonakis G, Simeonov V, Reinhorn A M, Buckle I G (1999): Implications of Spatial Variation of Ground Motion on the Collapse of Sr14/I5 Southbound Separation and Overhead Bridge in the Northridge Earthquake. *ACI International-Special Publication SP*, 187, 299-327.

[3] Miyachi K, Nakamura S, Manda A (2012): Progressive collapse analysis of steel truss bridges and evaluation of ductility. *Journal of Constructional Steel Research*, 78, 192-200.

[4] Zhou Y, Zhang L, Liu X (2005): Collapse simulation and analysis of Cypress viaduct during Loma Prieta Earthquake[J]. *Chinese Journal of Rock Mechanics and Engineering*, **24**(17):3035-3044.

[5] Bhattacharya S, Tokimatsu K, Goda K, Sarkar R, Shadlou M, Rouholamin M (2014): Collapse of Showa Bridge during 1964 Niigata earthquake: A quantitative reappraisal on the failure mechanisms. *Soil Dynamics and Earthquake Engineering*, 65, 55-71.

[6] Julian F D R, Hayashikawa T, Obata T (2007): Seismic performance of isolated curved steel viaducts equipped with deck unseating prevention cable restrainers. *Journal of Constructional Steel Research*, **63**(2), 237-253.

[7] Sun G, Luo R, Zuo Y, Li H, Du X (2014): Application of Multi-Scale Finite Element Modeling in Bridge Seismic Damage and Collapse Analysis. *In Challenges and Advances in Sustainable Transportation Systems* (pp. 580-587). ASCE.

[8] Mckenna F T (1997): *Object-oriented finite element programming: frameworks for analysis, algorithms and parallel computing* (*Doctoral dissertation, University of California, Berkeley*).

[9] Qu Z (2010): *Study on seismic damage mechanism control and design of rocking wall-frame structure*(*Doctoral dissertation,Tsinghua University, Beijing*).